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A MODEL-FOLLOWING TECHNIQUE FOR INSENSITIVE AIRCRAFT CONTROL SY--ETC(U)
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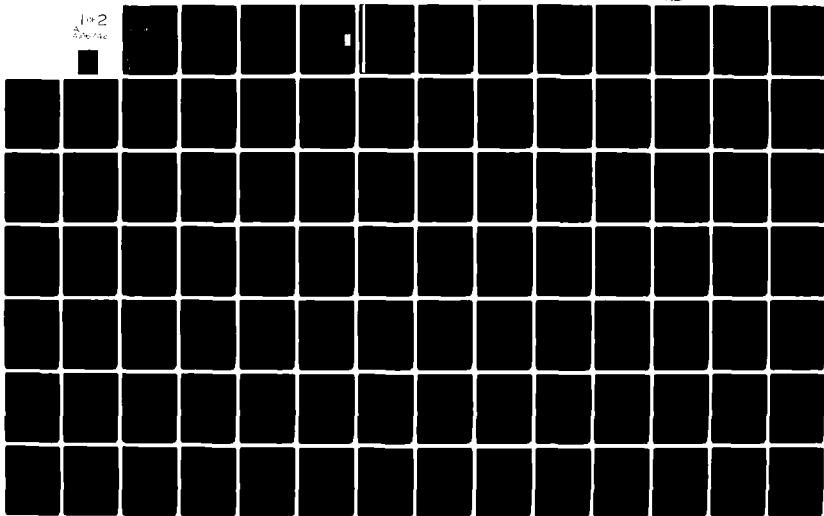
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ABSTRACT

The problem of designing multivariable control systems is addressed through the use of model-following methods. A number of different model-following techniques are discussed, and their advantages and disadvantages are presented. It is shown that model-following theories can provide useful structural insight to the designer, and that they can help to integrate the methods of "classical" and "modern" control. A new design method is presented which, when implemented, uses model following and full-state feedback to keep the dominant roots of a system constant. Under favorable circumstances, it can do this even in the presence of arbitrarily large parameter uncertainties. The method has the attractive feature that the parameter-insensitivity and disturbance-rejection characteristics of the system can be selected independently from the no-disturbance, nominal-plant performance. Application is made to several aircraft flight control problems.

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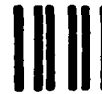
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**Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY**

**A MODEL-FOLLOWING TECHNIQUE FOR
INSENSITIVE AIRCRAFT CONTROL SYSTEMS**

by

George C. Nield IV

**Guidance and Control Laboratory
A Joint Facility of the
Department of Aeronautics & Astronautics
and the
Department of Mechanical Engineering**

**This Research was Supported by
The U.S. Air Force and
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LIST OF SYMBOLS

Scalars:

J	cost function
M_q	pitching moment due to pitch rate
p	roll rate
q	pitch rate
r	yaw rate
s	Laplace variable
α	angle of attack
β	sideslip angle
δ_a	aileron deflection
δ_e	elevator deflection
δ_f	flap deflection
δ_p	drag pedal deflection
δ_r	rudder deflection
δ_t	throttle position
θ	pitch angle
v	velocity
ϕ	roll angle

Vectors:

e	state error vector
u	control vector
x	state vector

y	output vector
z	augmented state vector
w	uncertain parameter vector

Matrices:

A	state weighting matrix
B	control weighting matrix
C	control gain matrix
F	state dynamics matrix
G	control distribution matrix
H	state selector or output matrix
I	identity matrix
S	state/control weighting matrix
0	zero matrix

Operators:

$\Delta(\cdot)$	increment or change in the following variable
-----------------	---

Subscripts:

$(\cdot)_m$	model
-------------	-------

Superscripts:

$\dot{(\cdot)}$	time derivative
$(\cdot)^t$	transpose
$(\cdot)^{-1}$	inverse

(•)* pseudo-inverse

Abbreviations:

ACF	Average Cost Function
deg	degrees
ft	feet
I.C.	initial condition
LQR	Linear Quadratic Regulator
MDEC	Modified Discrete Expected Cost
MFIC	Model Following for Insensitive Control
MGCC	Multistep Guaranteed Cost Control
OMF	Optimal Model Following
sec	seconds
SST	supersonic transport

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Chapter I

INTRODUCTION

1.1 BACKGROUND

"Model following" refers to a number of very general control-system design techniques which can be used to solve a wide variety of control-system problems. In the typical model-following problem, it is desired to force some or all of the states of a plant to "match", to a greater or lesser degree, the states of an ideal model. This model could be an actual hardware system; however, it is normally a mathematical model of the desired closed-loop plant behavior.

Two of the most common applications of model following have been in the area of aircraft flight control. The first type of problem involves the design of the flight control system for a new aircraft. Frequently, by referring to the Military Specifications for aircraft flying qualities, through simulator studies, or from past experience, the designers have established desired time-history responses to pilot commands. The objective is then to select the control-system structure and the control gains so that the aircraft will have responses which match the desired ones as closely as possible. Such a problem is ideally suited to the application of model-following techniques. In the second type of problem, we would like to make one kind of aircraft "feel" to the pilot like a different kind of aircraft. There are several reasons why we might

want to do this. For example, it is then possible to conduct systematic studies of different aircraft characteristics to determine their impact on handling qualities or the usefulness of new control modes. Also, it is often very helpful to be able to provide pilot training and/or initial evaluations for a new aircraft before the aircraft itself is available. By using special "variable-stability aircraft," preliminary testing has been accomplished on a number of recent development programs, such as the Supersonic Transport (SST), the F-16 fighter, and the Space Shuttle, long before the final prototypes were completed. Such efforts are classic examples of model-following problems since the goal is to make the first aircraft (the plant) act like the second aircraft (the model).

Although many papers have been written on the model-following problem, the question of how model following relates to control-system sensitivity has not been fully addressed. The purpose of this thesis is to review the various types of model-following techniques, to show how they are related to each other and to other design methods, and to show how model following can be used for designing insensitive controllers.

1.2 LITERATURE REVIEW

There are two ways to approach the actual implementation of a model-following system. In the first method, Implicit Model Following (also known as "Model in the Performance Index"), the model equations are only used during the design process to aid in the determination of the control gains. After the design is complete, the model is no longer

needed, and it is not necessary to implement the model equations in the actual system. The second method, known as Explicit Model Following (also called "Real Model Following" or "Model in the System"), requires the model equations to be incorporated as part of the control system. The model states are then assumed to be available, along with some or all of the plant states, for formulation of the control signals. Kreindler[16] and others have compared the explicit and implicit model-following methods for particular aircraft examples; however, conclusions on the overall merits of the two approaches have usually been rather subjective.

Whether a system uses an implicit model-following approach or an explicit one, it is clear that the ideal result would be to have the plant dynamics exactly match the model dynamics. Such a capability is referred to as "perfect model following." If perfect model following is possible (and the control gains are selected correctly), then when the plant and model are started at the same initial conditions and with no unknown disturbances, one or more of the plant states will identically match the corresponding model states. Erzberger[10] developed sufficient conditions and an expression for the control gains to obtain perfect model following for one class of implicit model-following systems, while Chan[5] and Morrow and Balasubramanian[20] gave the appropriate control laws for the explicit formulation. Asseo[3] and Motyka and Rynaski[22] extended some of these perfect-model-following ideas, while Curran[8] developed the related concept of "equicontrollability", and showed how perfect-model-following theories could be made more useful by allowing slight modifications of the model.

Tyler[29] examined the use of optimal control theory for solving model-following problems. He was followed by a number of researchers, including Kriendler[16], Gran, Berman, and Rossi[11], Kriechbaum and Stineman[17], Markland[19], Tiroshi and Elliot[27], and Yore[32]. Rediess and Whitaker[25] and Peled[24] also used models to formulate their optimal-control cost functions, but their methods were really parameter-optimization schemes which required the designer to determine the control system's structure and the appropriate form of any required compensation. Trankle and Bryson[28] developed a variation of earlier methods which they found useful for solving some classes of model-following problems.

Many of the above authors stated their "intuitive" feelings that explicit model-following systems had potential for reducing the sensitivity of control systems to parameter uncertainty. Nevertheless, there have been few if any attempts to quantify this capability, or to formulate an appropriate design method. Winsor and Roy[31] attempted to tackle the sensitivity issue, but their design method was complex and was limited to small parameter variations. In addition, they concluded that it was not possible to desensitize more than one variable at a time. Landau and Courtiol[18] apparently endorsed Winsor and Roy's findings and stated their belief that an adaptive system, even with its inherent complexity and nonlinearities, was the best approach. Kamiya[15] was probably the first to study the use of model-following systems specifically for parameter-insensitive control; however, he used a different structure than the one proposed in this work.

Since one of the major contributions of this work relates to parameter-insensitive controllers, it is important to mention briefly some of the other, non-model-following efforts in this area. Palsson and Whitaker[23] and Hadass[12] developed the technique of treating the uncertain parameters as random vectors. Harvey and Pope[13] and Vinkler[30] compared several different methods in their works, while Shenkar[26] and Ashkenazi[2] extended the most promising methods for the output-feedback case. In general, all of the methods proposed required large, time-consuming computer programs and used gradient-search algorithms to obtain the final results. Even so, the resulting systems had the unfortunate characteristic that the system performance and/or control use for the nominal system was typically degraded from what would have been obtained had the design been accomplished without regard to the parameter variations.

1.3 CHAPTER OUTLINES

Chapter II defines perfect model following and discusses how it relates to other methods such as pole placement. The criteria for achieving perfect model following are presented, and it is shown that they can provide useful insight into the structure of a control-system problem, even when other design methods are used.

Chapter III shows how we can combine model-following techniques and optimal control theory to take advantage of the respective strengths of "classical" and "modern" control. Both state-matching and dynamics-matching systems are developed, and the advantages and disadvantages of each type of system are presented.

Chapter IV begins with a review of past efforts in parameter-insensitive control. A new design method using model following is then presented which is shown to have significant advantages over previous techniques.

Chapter V gives conclusions and recommendations for future research.

1.4 SUMMARY OF CONTRIBUTIONS

1. A number of different model-following design techniques are evaluated, and their advantages and disadvantages and relationships to other methods are presented.
2. The relationship between feedback and feedforward control is demonstrated.
3. A new design method is presented which uses model following and full-state feedback to keep the dominant roots of a system constant. Under favorable circumstances, it can do this even in the presence of arbitrarily large parameter uncertainties. The method has the attractive feature that the parameter-insensitivity and disturbance-rejection characteristics of the system may be selected independently from the no-disturbance, nominal-parameter performance. The method is applicable to systems in which the plant can be made "well-damped" and "substantially faster" than the final system characteristics required by the model.

Chapter II

PERFECT MODEL FOLLOWING AS A MULTIVARIABLE DESIGN TOOL

2.1 REVIEW OF POLE-PLACEMENT TECHNIQUE

The use of full-state feedback, either by measurement or by estimation, gives the control-system designer great flexibility, since the roots of the closed-loop system's characteristic equation may be placed arbitrarily with the proper choice of control gains. This has led to a very simple, easy-to-use design technique for single-input, single-output systems. In this technique, known as pole placement, the characteristic equation of the desired closed-loop system is compared to the characteristic equation of the open-loop plant, modified so that there are feedback gains on all of the states. By matching the coefficients of the Laplace variable "s", a set of algebraic equations results which allows for solution of the control gains.

Let us take a simple, second-order system as an example. The differential equation of motion of an open-loop plant is given by $\ddot{x}+x=u$, or in state-space form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u.$$

Suppose the desired system has a natural frequency of 2 rad/sec and a damping ratio of .7. The desired characteristic equation is then

$$s^2 + 2.8s + 4 = 0.$$

Using full-state feedback, the control law is given by $u=Cx$, so we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1+c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

with a characteristic equation given by

$$s^2 - c_2s + 1 - c_1 = 0.$$

Comparing coefficients, we see that

$$-c_2 = 2.8$$

and

$$1 - c_1 = 4.$$

This yields

$$c_1 = -3$$

and

$$c_2 = -2.8.$$

Unfortunately, the technique breaks down for multiple inputs and outputs. In the above example, there are two degrees of freedom from the two control gains, and two equations resulting from comparing the coefficients of "s". However, with additional controls, additional degrees of freedom are added through additional control gains, but there are no additional equations, thus preventing a unique solution. Suggestions for circumventing this problem include arbitrarily setting some of the control gains equal to zero, or assuming that some of the entries in the dynamics matrix are small enough to be neglected. In either case, several iterations are usually required, since there is no guarantee that the resulting system will have desirable response characteristics.

2.2 DERIVATION OF PERFECT-MODEL-FOLLOWING CONTROL

Suppose that rather than matching the characteristic equations, we try to match the entire dynamics matrices. Repeating Erzberger's derivation[10], but with our notation, we let the plant be described by

$$\dot{x} = Fx + Gu$$

$$y = Hx.$$

The model, which describes the desired system behavior, is given by

$$\dot{x}_m = F_m x_m.$$

For y and x_m to have the same dynamics, we require that

$$\dot{y} = F_m y = F_m H x.$$

Also,

$$\dot{y} = H \dot{x} = H F x + H G u.$$

Thus, we have

$$H F x + H G u = F_m H x.$$

Solving for u , we obtain

$$u = [(H G)^t (H G)]^{-1} (H G)^t (F_m H - H F) x.$$

We define the pseudo-inverse of $H G$ as:

$$(HG)^* = [(HG)^t(HG)]^{-1}(HG)^t.$$

Then,

$$u = (HG)^*(F_m H - HF)x.$$

By substituting this expression for u into the equation for \dot{y} , we get

$$\dot{y} = HFx + (HG)(HG)^*(F_m H - HF)x.$$

Then, in order for \dot{y} to equal $F_m y$, we must have

$$HFx + (HG)(HG)^*(F_m H - HF)x = F_m Hx$$

$$[HF + (HG)(HG)^*(F_m H - HF) - F_m H]x = 0$$

$$(HG)(HG)^*(F_m H - HF) - I(F_m H - HF) = 0$$

$$[(HG)(HG)^* - I][F_m H - HF] = 0.$$

The final equation above, known as Erzberger's condition, gives a sufficient condition for exactly matching the dynamics of the plant and the model for this class of problem. Such matching is known as "perfect model following".

We can generalize the above procedure to handle inputs to the model, as well as the matching of only a subset of model states. Let the plant again be described as

$$\dot{x} = Fx + Gu$$

$$y = Hx.$$

The desired system dynamics are given by the model equations:

$$\dot{x}_m = F_m x_m + G_m u_m$$

$$y_m = H_m x_m.$$

We would like to have

$$\dot{y} = \dot{y}_m.$$

Thus,

$$HFx + HGu = H_m F_m x_m + H_m G_m u_m$$

$$HGu = H_m F_m x_m - HFx + H_m G_m u_m$$

$$u = (HG)^{-1} H_m F_m x_m - (HG)^{-1} HFx + (HG)^{-1} H_m G_m u_m.$$

Or,

$$u = C_1 x + C_2 x_m + C_3 u_m,$$

where

$$C_1 = -(HG)^*HF$$

$$C_2 = (HG)^*H_m F_m$$

and

$$C_3 = (HG)^*H_m G_m.$$

Notice that this general expression for the perfect-model-following control contains three terms: a feedback of plant states x , a feedforward of model states x_m , and a feedforward of model inputs u_m . However, this does not mean that we are restricted to this type of structure. If we are able to successfully make a plant state exactly follow a model state, then obviously the two signals are equivalent, and we can substitute one of them for the other in the control law. For example, suppose we are working a problem in which the number of states to be matched is equal to the number of model states (i.e., H_m is square). Then H_m^{-1} will normally exist.

Since $Hx = H_m x_m$,

$$x_m = H_m^{-1} Hx.$$

The perfect-model-following control law can then be written as

$$\begin{aligned} u &= (HG)^* H_m F_m H_m^{-1} Hx - (HG)^* H Fx + (HG)^* H_m G_m u_m \\ &= (HG)^* [H_m F_m H_m^{-1} H - HF]x + (HG)^* H_m G_m u_m. \end{aligned}$$

Thus for this case, we can achieve perfect model following with only feedback from the plant states, without having to implement the model system. For $H_m = I$ and $G_m = 0$, we are left with

$$u = (HG)^* [F_m H - HF]x,$$

which is the control law derived by Erzberger.

If the number of states to be matched is equal to the number of plant states (i.e., H is square), then H^{-1} will normally exist.

Thus,

$$x = H^{-1} H_m x_m$$

and

$$u = (HG)^* H_m F_m x_m - (HG)^* H F H^{-1} H_m x_m + (HG)^* H_m G_m u_m$$

$$= (HG)^*[H_m F_m - H F H^{-1} H_m]x_m + (HG)^*H_m G_m u_m.$$

This control law can therefore be implemented with only feedforward from the model states, without the need for measuring any plant states.

Finally, if all of the plant states are matched to all of the model states, then both H and H_m are square and invertible, and either of the above control laws can be used. As a result, it is possible to implement the control system with either feedforward or feedback, or a combination of the two.

By substituting the general expression for u into the equation for \dot{y} , we get

$$\dot{y} = H F x + (HG)(HG)^*[H_m F_m x_m - H F x] + (HG)(HG)^*H_m G_m u_m.$$

We would also like to have

$$\dot{y} = H_m F_m x_m + H_m G_m u_m.$$

Equating the two expressions,

$$H F x + (HG)(HG)^*[H_m F_m x_m - H F x] + (HG)(HG)^*H_m G_m u_m = H_m F_m x_m + H_m G_m u_m$$

$$(HG)(HG)^*[H_m F_m x_m - H F x] + [H F x - H_m F_m x_m]$$

$$+ (HG)(HG)^*H_m G_m u_m - H_m G_m u_m = 0$$

$$[(HG)(HG)^* - I][H_m F_m x_m - HFx] + [(HG)(HG)^* - I]H_m G_m u_m = 0.$$

Since u_m is arbitrary, we must require that

$$[(HG)(HG)^* - I][H_m F_m x_m - HFx] = 0$$

and

$$[(HG)(HG)^* - I]H_m G_m = 0.$$

The above conditions give the general criteria for perfect model following to exist. By examining specific cases, it is possible to use these relations to get meaningful conditions. For example, if $H_m=I$ and $G_m=0$, we get

$$[(HG)(HG)^* - I][F_m x_m - HFx] = 0.$$

Since $x_m = H_m^{-1} Hx = Hx$, we can write

$$[(HG)(HG)^* - I][F_m H - HF]x = 0.$$

Therefore,

$$[(HG)(HG)^* - I][F_m H - HF] = 0,$$

which is Erzberger's condition.

2.3 INTERPRETATION OF CONDITIONS FOR PERFECT MODEL FOLLOWING

It is interesting to note that a sufficient condition for perfect model following to exist is

$$[(HG)(HG)^* - I] = 0.$$

First, let us define the term "directly controllable" to mean that a state has at least one non-zero entry in the corresponding row of the control matrix, G. For example, in the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,$$

x_2 is directly controllable, while x_1 is not. The fact that a state is directly controllable therefore means that it is possible to modify that row of the dynamics matrix. With this definition, the above condition can be understood as requiring that each plant state to be matched must be directly controllable, and that the number of controls be equal to the number of plant states to be matched. The reason that this is a sufficient condition and not a necessary one relates to the fact that even if a state is not directly controllable, sometimes the row of the

plant dynamics matrix is already the same as the corresponding row in the model dynamics matrix, so no additional control effort is required to force the match.

Even though the perfect-model-following gains are straightforward and easy to calculate, it might at first appear that the criteria to be satisfied are quite stringent, and that perfect model following is therefore rather limited in use. However, with some slight modifications, the theory can be extended to cover a much broader range of problems.

To restate the sufficient condition for perfect model following:

Plant states can be made to match model states exactly if they are directly controllable and if the number of controls is equal to the number of states to be matched.

If the number of controls is greater than the number of states to be matched, one can simply delete the extra controls. If the number of controls is less than the number of states to be matched, then the mathematics does yield a solution, but it represents only an approximate match of the plant and model states. Actually, it is a "least-squares" compromise of the desired matches. Whether this represents an acceptable solution depends upon how different the open-loop plant and model dynamics are.

In many aircraft, actuators are used to drive the control surfaces. For such systems, the primary plant states are not "directly controllable" as defined above, thus precluding the straightforward application of perfect-model-following theory. Fortunately however, the dynamics of aircraft control actuators are usually much faster than the vehicle dynamics. As a result, they can normally be neglected for the initial control-system designs.

Even if a plant state is not directly controllable, it will still match a model state if its dynamics are the same as the model state, and the state(s) through which it is indirectly controlled are made to match the model. For example, in aircraft equations of motion, roll angle is normally just the integral of roll rate for both the plant and the model. As a result, even though roll angle is not a directly controllable state, it will match the model if roll rate is made to match (assuming the initial conditions are the same). If we have a situation where a plant state is not directly controllable and there are no corresponding higher-order states in the model, then it will be necessary to cancel the extra plant states in order to get an exact match. For example, the only way to get a $1/s^2$ system to behave like a $1/s$ model is to somehow get rid of an "s", such as by adding a differentiator. With a noisy system, a solution of this type would probably not be practical. Alternatively, as Curran has shown[8], the plant can be made to match a modified model, which consists of the original model plus a number of additional arbitrarily fast roots. In other words, we could have our $1/s^2$ system behave like a model with dynamics given by $1000/s(s+1000)$. Of course, the faster the additional roots are (and hence the closer the

modified model is to the original one), the greater the control gains and control effort which will be required.

Even if the actual selection of control gains in a system is done by a different method, applying perfect-model-following philosophy can often allow the control-system designer to gain some helpful structural insight into the problem. For example, the number of controls should be based on the number of states for which a particular time response is required (as opposed to simply being stable). Even though the standard control-theory criteria may indicate that all of the states of a system are "controllable" with only a single control, if it is desired to specify precisely the responses of more than one state, additional controls will probably be necessary. Using an aircraft as an example, if the elevator is the only longitudinal control, then it is only possible to guarantee a particular response for a single output variable - whether it be pitch rate, or normal acceleration, or a linear combination of the two. If it is desired to specify the responses for both pitch rate and normal acceleration, then a second longitudinal control, such as a canard or maneuver flaps, would be required. Along the same lines, the control-system designer should strive to have each state of interest be directly controllable. If this is not possible because an actuator is in the system, the control-system designer should push for as "fast" an actuator as possible. This not only makes the designer's job easier, it also allows him to design a control system which will provide the maximum flexibility and best total performance. In the end of course, cost or other constraints may not permit the control-system designer to have the final say.

2.4 EXAMPLE: LATERAL-DIRECTIONAL DECOUPLING OF A T-33 AIRCRAFT

As an example of the application of perfect-model-following theory, consider the problem of decoupling the lateral-directional axes of a T-33 aircraft. This example has been solved by geometric methods in [6] and with an explicit model-following technique in [20].

Lateral-directional decoupling refers to the desire to make the yaw and sideslip motions of an aircraft independent from its rolling motions. If this can be accomplished, the pilot workload is significantly reduced, resulting in improved safety for commercial operations or greater weapons-delivery accuracy for military aircraft. The particular aircraft for this example is a T-33 which has been modified for flying-qualities research. In addition to the normal rudder and aileron control surfaces, it has hydraulically operated "drag petals" on the wing-tip fuel tanks. These surfaces can be differentially extended in flight in order to generate significant yawing moments, if required.

The equations of motion of the unaugmented T-33 are given by

$$\dot{x} = Fx + Gu,$$

where

$$x^t = [p \ \phi \ r \ \beta]$$

$$F = \begin{bmatrix} -3.18 & 0 & .63 & -10.6 \\ 1. & 0. & 0. & 0. \\ -.06 & 0. & -.27 & 4.18 \\ .022 & .0644 & -.998 & -.151 \end{bmatrix}$$

$$G = \begin{bmatrix} -14.4 & 1.5 & 0. \\ 0. & 0. & 0. \\ 0. & -2.59 & -.96 \\ 0. & .037 & 0. \end{bmatrix}$$

and

$$u^t = [\delta a \quad \delta r \quad \delta p].$$

The state vector consists of roll rate, roll angle, yaw rate, and sideslip angle, while the control vector is made up of aileron position, rudder position, and drag pedal position.

The model equations were obtained by replacing the undesirable coupling terms in the plant equations with zeros. The resulting model is described by

$$\dot{x}_m = F_m x_m + G_m u_m,$$

with

$$x_m^t = [p_m \quad \phi_m \quad r_m \quad \beta_m]$$

$$F_m = \begin{bmatrix} -3.18 & 0. & 0. & 0. \\ 1. & 0. & 0. & 0. \\ 0. & 0. & -.27 & 0. \\ 0. & 0. & 0. & -.151 \end{bmatrix}$$

$$G_m = \begin{bmatrix} -14.4 & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & -.96 \\ 0. & .037 & 0. \end{bmatrix}$$

and

$$u_m^t = [6a_m \ 6r_m \ 6p_m].$$

Since there are three controls, and the roll rate, yaw rate, and sideslip angle are all "directly controllable," we are guaranteed that all three of these states can be made to match the model states exactly. In addition, since roll angle is simply the integral of roll rate, it will also match if roll rate is made to match. Thus, we are able to match exactly all four plant states to the four model states. Note that this system may be implemented either with feedforward or with feedback. The plant-state-feedback or model-state-feedforward gains are

$$C_1 \text{ or } C_2 = \begin{bmatrix} -.0619 & -.1813 & 2.8534 & -.7361 \\ -.5946 & -1.7405 & 26.9730 & 0. \\ 1.5417 & 4.6958 & -72.7708 & 4.3542 \end{bmatrix}.$$

The gains on the model inputs are

$$C_3 = \begin{bmatrix} 1. & .1042 & 0. \\ 0. & 1. & 0. \\ 0. & -2.6979 & 1. \end{bmatrix}$$

As expected, the closed-loop dynamics matrix using these gains exactly matches the model dynamics matrix.

2.5 EXAMPLE: LATERAL-DIRECTIONAL CONTROL OF A B-26 AIRCRAFT

In the previous example, the plant had three controls and three directly-controllable states, which made it possible to match exactly all of the model states. However, often the designer will be faced with a problem with fewer controls than states. Such a case is examined here. The plant equations represent the lateral-directional equations of motion of a B-26 aircraft:

$$\dot{x} = Fx + Gu,$$

with

$$x^t = [\phi \quad p \quad \beta \quad r]$$

$$F = \begin{bmatrix} 0. & 1. & 0. & 0. \\ 0. & -2.93 & -4.75 & -.78 \\ .086 & 0. & -.11 & -1. \\ 0. & -.042 & 2.59 & -.39 \end{bmatrix}$$

$$G = \begin{bmatrix} 0. & 0. \\ 0. & -3.91 \\ .035 & 0. \\ -2.53 & .31 \end{bmatrix}$$

and

$$u^t = [6r \ 6a].$$

The state vector consists of roll angle, roll rate, sideslip angle, and yaw rate, while the control vector is made up of rudder position and aileron position.

The model was determined from simulator studies to provide improved aircraft handling qualities:

$$\dot{x}_m = F_m x_m + G_m u_m.$$

with

$$x_m^t = [\phi_m \ p_m \ \beta_m \ r_m]$$

$$F_m = \begin{bmatrix} 0. & 1. & 0. & 0. \\ 0. & -1. & -73.14 & 3.18 \\ .086 & 0. & -.11 & -1. \\ .0086 & .086 & 8.95 & -.49 \end{bmatrix}$$

$$G_m = G$$

and

$$u_m^t = [\delta r_m \ \delta a_m].$$

If we try to apply Erzberger's condition for perfect model following, we find that it is not satisfied; thus both Erzberger[10] and Chan[5] concluded that "perfect model following is not possible." However, both authors proceeded to solve for the perfect-model-following gains, and found that since the plant and model equations were not too different, a reasonably close "compromise" match of the four states could be achieved.

The gains for this case were determined to be

$$C_1 \text{ or } C_2 = \begin{bmatrix} -.0034 & -.1110 & -.3706 & -.0846 \\ 0. & -.4936 & 17.4911 & -1.0128 \end{bmatrix}.$$

Since the plant and model control matrices are the same, the gain matrix from the model inputs, C_3 , is simply the identity matrix.

In this particular problem, the plant and model equations were quite similar, and the compromise solution which attempted to match all four states is probably acceptable. Obviously, this will not always be the case. Therefore, it is important to note that by using the generalized perfect-model-following control law, it is possible to achieve perfect model following for a subset of the plant states. Since there are two controls, we are guaranteed to be able to match two states; however, since roll angle is simply the integral of roll rate, it will be matched if roll rate is matched. If we want to match roll angle, roll rate, and yaw rate, the selector matrix becomes

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The resulting feedback gain matrix for the plant states is

$$C_1 = \begin{bmatrix} 0. & -.1084 & .8749 & -.1786 \\ 0. & -.7494 & -1.2148 & -.1995 \end{bmatrix},$$

while the feedforward gain matrix for the model states is

$$C_2 = \begin{bmatrix} -.0034 & -.0027 & -1.2455 & .0940 \\ 0. & .2558 & 18.7059 & -.8133 \end{bmatrix}.$$

Roll-angle time histories of the 4-state "compromise" match and the 3-state "exact" match are given in Figure 1. The results for roll rate and yaw rate are similar, with the 3-state-match design duplicating the model system exactly. Although we do not have any guarantees as to what kind of response we will get for the state we did not try to match (sideslip angle), in this particular problem it is very close to the model response.

Obviously, other choices of H are possible if it is desired to match a different subset of the states. Essentially, this control law uses the feedback from the plant states to cancel the plant dynamics and the feedforward from the model states to obtain the desired responses. Note that using Erzberger's control law with this type of problem will not yield exact matches, even with the same selector matrix as the one used here.

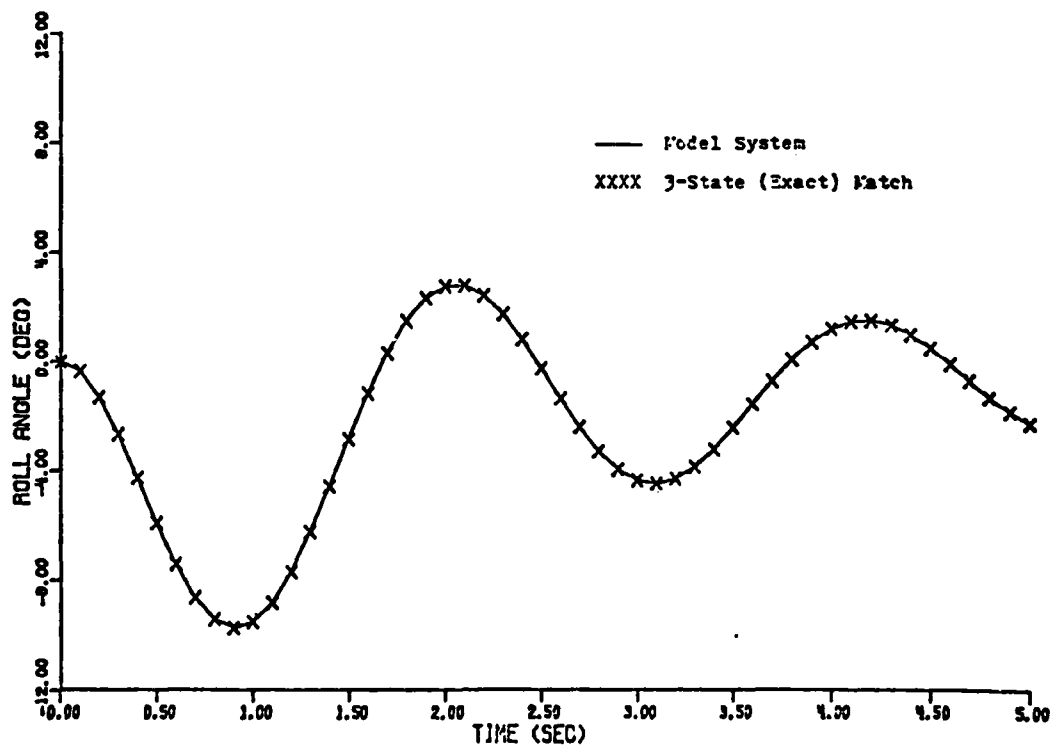
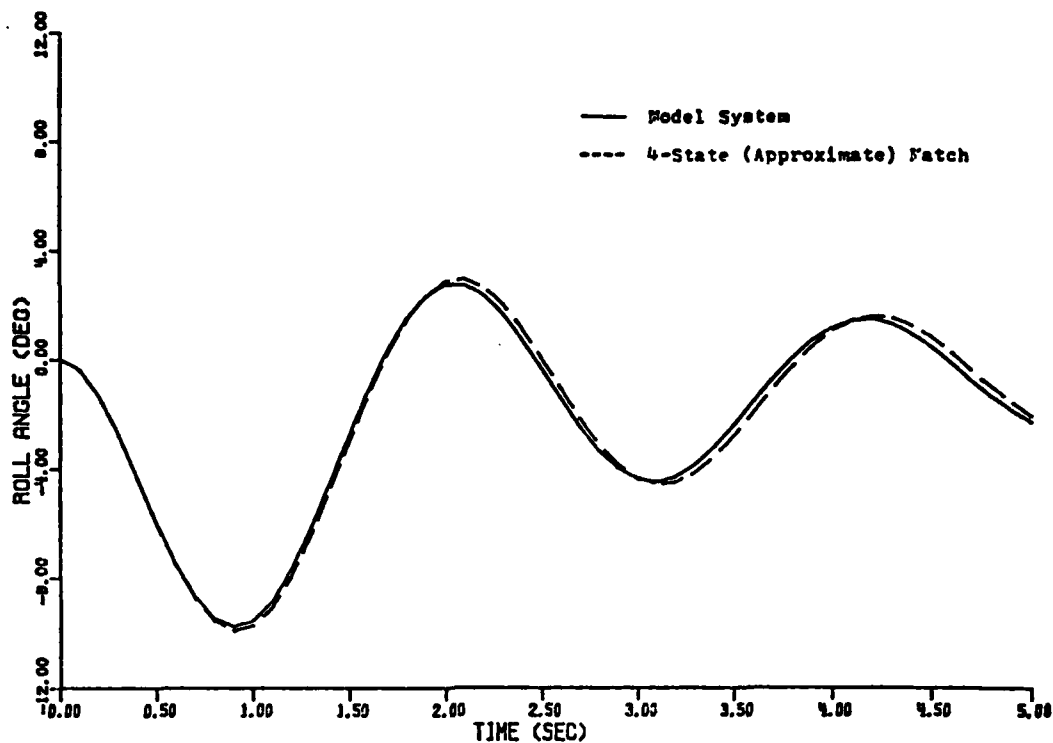


Figure 1: Comparison of B-26 Roll-Angle Responses

Chapter III

MODEL FOLLOWING AND OPTIMAL CONTROL

3.1 SELECTION OF WEIGHTING MATRICES FOR OPTIMAL CONTROL PROBLEMS

Many control-system problems can be formulated in such a way that it is desired to find the control gains which will minimize a cost function of the form

$$J = \int_0^{\infty} (x^t A x + u^t B u) dt,$$

where A and B are weighting matrices which penalize state excursions and control usage respectively. The mathematics for the solution of such "optimal control" problems is well known, and is easily accomplished on a digital computer. The resulting control laws have a number of favorable properties, such as guaranteed stability. In addition, because of the matrix structure, multivariable problems are no more difficult to solve than single-input, single-output ones. Nevertheless, optimal control techniques have not seen widespread acceptance in industry, particularly for aircraft control problems. Part of the reason for this may lie in the relative newness of the optimal control methods, and in the availability of skilled classical designers whose experience and insight

allow them to obtain excellent results with Root-Locus or Bode-Plot techniques, even for very complicated problems. However, some of the reluctance to use the modern control methods may be due to the difficulty of selecting the weighting matrices A and B in order to obtain the desired results. Increasing the weighting on a state tends to speed up the response of that state, while increasing the weighting on a control tends to cut down on the use of that control. However, it is sometimes difficult to translate such specifications as frequency, damping, rise time, or overshoot into appropriate choices for the weighting matrices. Thus, many design iterations may be required before acceptable results are obtained, if the system requirements are ones of this type. As with classical design techniques, a great deal of experience and insight is found to be helpful, if not required, to achieve the desired results.

In an attempt to provide some guidance for the selection of the weighting matrices, Bryson's Rule was formulated[4]. Bryson's Rule states that A and B should be diagonal matrices with elements equal to the inverse of the square of the maximum allowable state excursions for A, and the inverse of the square of the maximum allowable control usage for B. Although such a procedure has been shown to provide a reasonable first guess for the matrices, it by no means eliminates the trial-and-error process. In addition, it is really designed to handle the steady-state regulator problem rather than the meeting of transient-response specifications.

A different approach to the problem is known as destabilization. In this method, the cost function is modified by the multiplication of the

factor $\exp(\alpha t)$, with $\alpha > 0$. Since the optimal control solution is guaranteed to be stable, the resulting closed-loop roots are "stabilized" or "shifted to the left" in the s-plane, by an amount α . The problem with this approach is that all of the roots are shifted, even the ones which already have an acceptable response. Thus, control effort is wasted, and some variables end up being over-damped and/or faster than required.

By combining the techniques of model following and optimal control, it is possible to take advantage of the power and flexibility of the optimal control methods while also incorporating time-response information, resulting in a more straightforward design procedure. This procedure, sometimes known as "Optimal Model Following", may be accomplished using either of two methods: state matching or dynamics matching.

3.2 STATE MATCHING

Let a plant be described by

$$\dot{x} = Fx + Gu$$

$$y = Hx,$$

with the desired system performance being given by the model equations

$$\dot{x}_m = F_m x_m$$

$$y_m = H_m x_m.$$

The H and H_m matrices are "selector" matrices which identify which states are to be matched. Since we would like to have y be as close as possible to y_m , we attempt to minimize the cost function

$$J = \int_0^{\infty} [(y_m - y)^t A (y_m - y) + u^t B u] dt,$$

with the system dynamics being given by

$$\begin{bmatrix} \dot{x} \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} F & 0 \\ 0 & F_m \end{bmatrix} \begin{bmatrix} x \\ x_m \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} u.$$

If we define a new state variable

$$z = \begin{bmatrix} x \\ x_m \end{bmatrix},$$

then the cost function may be rewritten as

$$J = \int_0^{\infty} (z^t \tilde{A} z + u^t B u) dt,$$

where

$$A' = \begin{bmatrix} H^t A H & -H^t A H_m \\ -H_m^t A H & H_m^t A H_m \end{bmatrix}.$$

The resulting control law will be

$$u = Cz = C_1 x + C_2 x_m,$$

and therefore requires both plant states and model states to implement it.

With the state-matching approach, it is the "speed" of the plant roots which is determined by the choice of weighting matrices in the cost function. By making these roots fast enough, the plant transfer function approaches unity. As a result, the plant outputs will accurately "follow" any input signal. In this case, the input signal is simply the output of our model system.

Unfortunately, if the plant roots are not made sufficiently fast, either because of control limitations or for other reasons, they may become the dominant factor in the overall system response, overshadowing the model roots. Thus, even if good steady-state performance is achieved, the resulting transient response may be totally different from the desired one. In addition, since it is not possible to have the

plant output exactly follow the input unless the transfer function is identically one, we would not be able to achieve perfect model following with this type of system unless the plant became infinitely fast.

3.3 DYNAMICS MATCHING

Suppose that instead of trying to minimize the error between the plant and model states, we attempt to match the dynamics. Given a plant

$$\dot{x} = Fx + Gu$$

$$y = Hx,$$

and a model

$$\dot{x}_m = F_m x_m,$$

we would like the dynamics of y to match the dynamics of x_m . In other words, we want \dot{y} to be as close as possible to $F_m y$. To accomplish this we can minimize the cost function

$$J = \int_0^{\infty} [(\dot{y} - F_m y)^T A (\dot{y} - F_m y) + u^T B u] dt,$$

which is equal to

$$\int_0^{\infty} [(HFx + HGu - F_m Hx)^t A (HFx + HGu - F_m Hx) + u^t B u] dt.$$

Multiplying out and regrouping terms, we get

$$J = \int_0^{\infty} \{ x^t [(HF - F_m H)^t A (HF - F_m H)x + 2x^t [(HG)^t A (HF - F_m H)]^t u + u^t [(HG)^t A (HG) + B] u \} dt.$$

Defining

$$\hat{A} = [(HF - F_m H)^t A (HF - F_m H)]$$

$$\hat{S} = [(HG)^t A (HF - F_m H)]$$

$$\hat{B} = [(HG)^t A (HG) + B],$$

we can write

$$J = \int_0^{\infty} (x^t \hat{A} x + 2x^t \hat{S}^t u + u^t \hat{B} u) dt.$$

If the cross-product term could be made to disappear, the problem would then be in the standard optimal control form. Kriendler[16] points out that frequently the plant being worked on will contain actuators, which isolate the plant states of interest from the control inputs. In such cases, $HG=0$. Therefore, $\hat{S}=0$ and $\hat{B}=B$, and we are left with

$$J = \int_0^{\infty} (x^t \hat{A} x + u^t B u) dt,$$

which is easily solved in the standard way. However, with a little additional manipulation (as suggested by Anderson and Moore[1]), even this restriction can be removed. Completing the square,

$$\begin{aligned} J &= \int_0^{\infty} (u^t \hat{B} u + 2x^t \hat{S}^t u + x^t \hat{S}^t \hat{B}^{-1} \hat{S} x - x^t \hat{S}^t \hat{B}^{-1} \hat{S} x + x^t \hat{A} x) dt \\ &= \int_0^{\infty} [(u + \hat{B}^{-1} \hat{S} x)^t \hat{B} (u + \hat{B}^{-1} \hat{S} x) + x^t (\hat{A} - \hat{S}^t \hat{B}^{-1} \hat{S}) x] dt \\ &= \int_0^{\infty} (x^t \bar{A} x + u_1^t \hat{B} u_1) dt, \end{aligned}$$

where

$$\bar{A} = \hat{A} - \hat{S}^t \hat{B}^{-1} \hat{S}$$

and

$$u_1 = u + \hat{B}^{-1} \hat{S} x.$$

It is interesting to examine the case when perfect model following is possible (as discussed in Chapter II), and no weight is put on the control. For this problem \bar{A} becomes 0, which in turn means that no weighting will be assigned to the states. As a result, the output of an optimal control program will be zero gains for u_1 (assuming the system was stable to start with). If we let A be the identity matrix (i.e., each state match is equally important), then

$$\hat{B} = [(HG)^t(HG)]$$

$$\hat{S} = [(HG)^t(HF - F_m H)],$$

and the control is given by

$$u = -[(HG)^t(HG)]^{-1} (HG)^t (HF - F_m H) x$$

$$= (HG)^t (F_m H - HF) x,$$

the same results which were achieved in Chapter II, although admittedly, this time after a much more complex procedure. Thus, with the dynamics-matching approach, we can achieve perfect model following when it is possible, without the need for an infinitely fast plant.

3.4 GENERAL REMARKS

In the preceeding sections it was shown how the ideas of model following and optimal control could be combined to produce "Optimal Model Following" design methods. With regular optimal control, selection of the weighting matrices is a "black art" at best. Putting too little weight on the control results in ridiculously fast roots and impractically high control levels, and changing the weighting on one state usually changes the response of other variables as well. With Optimal Model Following, the designer need only formulate a model whose dynamic equations satisfy the time response specifications. Typically, we might then let A be the identity matrix (in other words, all state matches are equally important), and let B equal a very small number. If perfect model following is possible and the dynamics-matching method is used, B can even be set to zero. In this case the control gains will not increase past the finite gains which are required for the match, no matter what A is, and the plant states will exactly match the model states. Kreindler[16] noted that with an actuator in the plant but not in the model, use of a small B led to high feedback gains on the actuator, essentially trying to "speed up" the actuator root. This makes sense when we consider the perfect-model-following ideas of Chapter II. With an actuator in the system, some of the plant states are not directly

controllable, so perfect model following is not possible, in general. Closer and closer matches can be achieved by speeding up the response of the state through which the plant states are controlled (i.e. the actuator). However, because of nonlinearities or actuator saturation, there is usually a physical limit to the actuator speed of response. In any case, whether perfect model following is possible or not, the designer's task is a very straightforward one. Even if some iteration is required, it is always obvious which direction to proceed. If the control use is too high, just increase the B matrix. If the degree of matching is too low, just decrease B. In this way the designer can easily achieve a balance between control use and the "goodness" of the match of the desired time response. The method thus serves as a kind of link between the time-response ideas of classical control theory, and the cost function and "control-oriented" approach of optimal control theory.

What are some of the problems or shortcomings of these methods? An obvious disadvantage of all explicit model-following systems is the necessity to implement the model equations. This requirement means that the control computer will probably have to be larger and faster than it otherwise would have to be. On the other hand, as we will see in Chapter IV, explicit model-following systems have advantages in disturbance rejection and control-system sensitivity. (State-matching systems require an explicit formulation, while dynamics-matching systems may be either explicit or implicit.) If a plant state is not directly controllable, attempts to match the model dynamics closely will result in a speeding up of the transfer state (actuator) according to how heavy the weighting on the match was made. Finally, because the objective in

optimal control problems is to minimize a cost function, there will always be an answer, even if it is totally unacceptable. Thus, the use of very ambitious models without the willingness to expend large amounts of control, or the attempt to match many states with only a few controls, will likely yield disappointingly poor matches. As a result, it is probably a good idea to review the perfect-model-following criteria to get a feel for what is possible before determining the objectives. And, once a final design has been arrived at, system testing or simulation is a necessity for verifying the performance of complex systems.

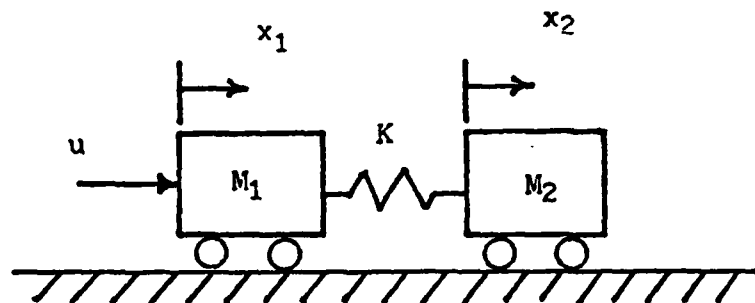
3.5 EXAMPLE: TWO MASSES AND A SPRING

As an example of the ease of the Optimal Model Following design process, specifically the dynamics-matching method, consider the system consisting of two masses and a spring shown in Figure 2.

The single control acts only on the first mass, but it is desired to control the motion of the second mass.

The plant equations of motion are given by

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u.$$



$$M_1 = M_2 = K = 1$$

Figure 2: Two Masses and a Spring

We would like the second mass to behave like a second-order system with a frequency of 2 rad/sec and a damping ratio of .7. The model is therefore described by

$$\begin{bmatrix} \dot{x}_m \\ \ddot{x}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -2.8 \end{bmatrix} \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix}$$

If we were trying to solve this problem with the standard Linear Quadratic Regulator (LQR) approach, we might initially try using an A

matrix with unity weighting on the position of the second mass and zeroes elsewhere. We could then try several different values of B to see what would happen. A plot of the resulting root locations is given in Figure 3.

Since the cost function does not contain any direct information about the desired transient response, it would have been a lucky coincidence if the plant roots had ended up close to the model roots. It is no doubt possible to do much better than we have done here, given enough tries. We can increase the weighting on one of the states to speed up that state, or increase the weighting on the derivative of a state to increase its damping. However, this is clearly a trial-and-error process, and it may take a large number of attempts before satisfactory results are achieved.

If we use Optimal Model Following, the design process is greatly simplified. Since we are trying to match the third and fourth plant states to the model, the selector matrix is

$$H = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The results are shown in Figure 4.

Lowering the weighting on the control causes two of the plant roots to proceed rapidly toward the model roots, while the remaining plant

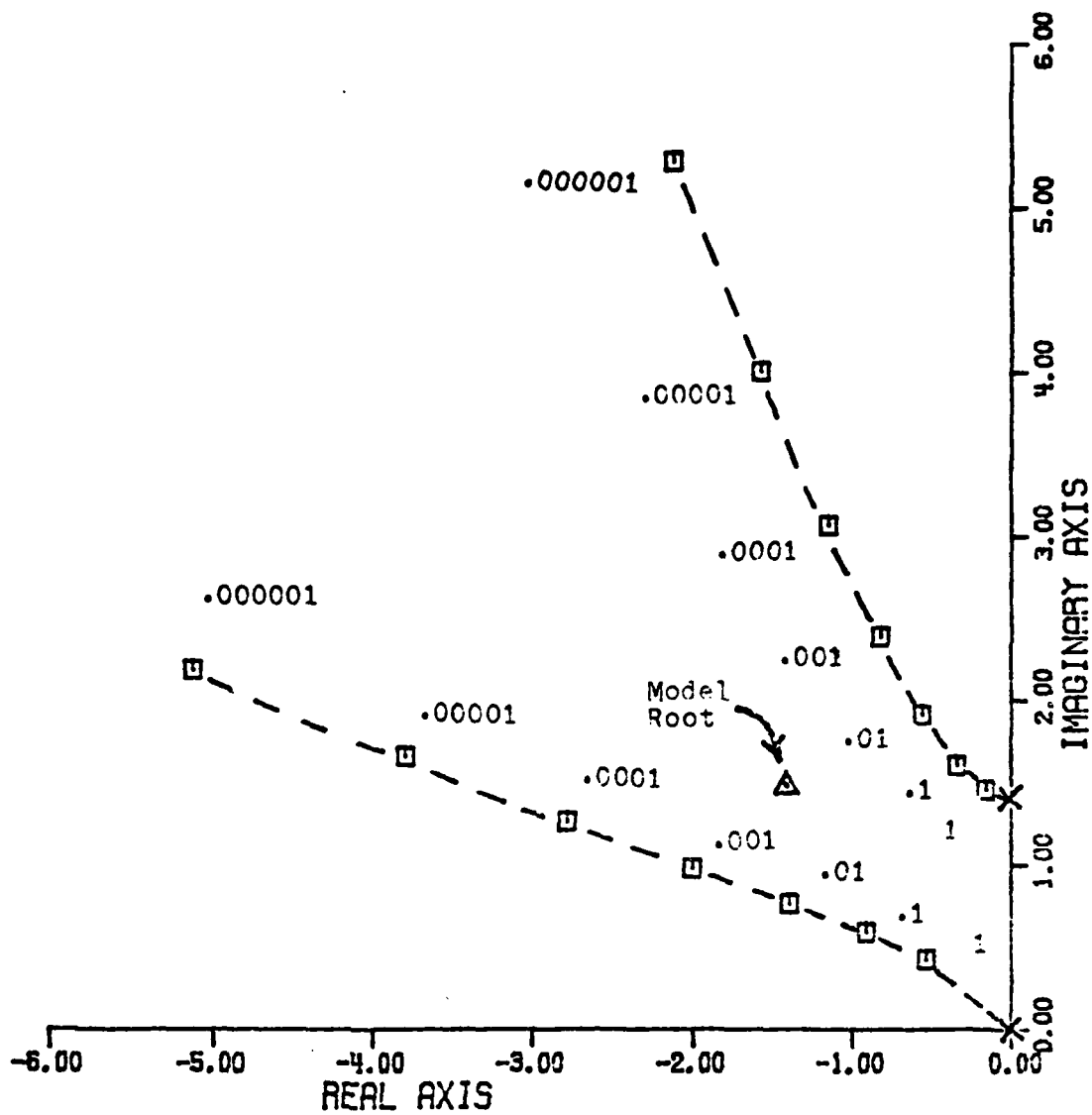


Figure 3: Locus of Roots v.s. Control Weighting - LQR Design

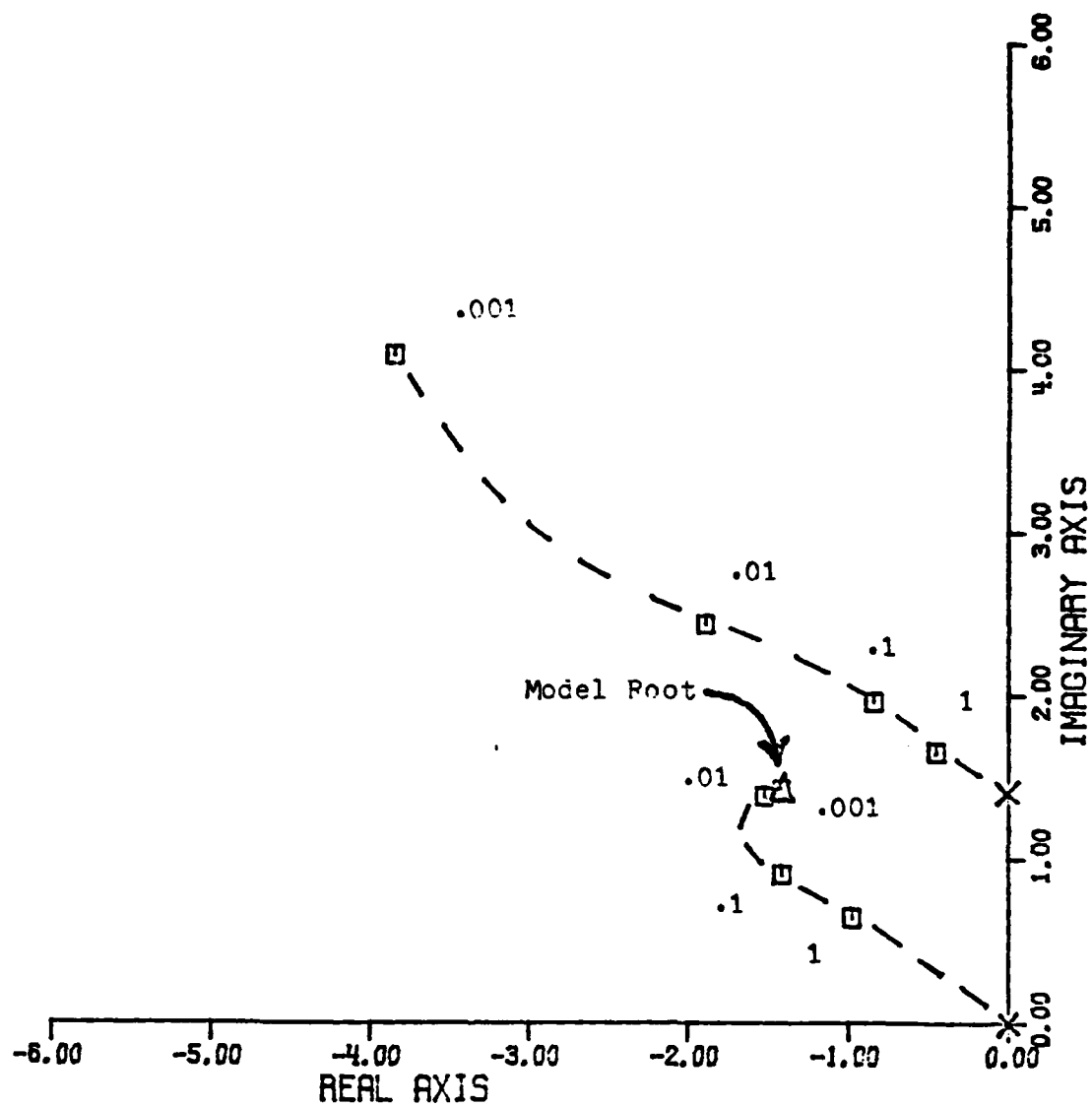


Figure 4: Locus of Roots v.s. Control Weighting - OMF Design

roots become very fast. Since the second mass is not directly controllable, the first mass acts like an actuator and becomes faster and faster as the weighting on the control is decreased. The designer needs only to decide upon the maximum acceptable control use, which will then determine how closely the plant states will come to achieving the desired time response. It should be noted that Optimal Model Following is not a pole-matching technique, and therefore the plant roots may not always proceed directly to the model roots. However, the time responses of the plant states will get closer and closer to the time responses of the model as the control weighting is decreased. A plot of the impulse response of the second mass with an LQR design is given in Figure 5, while the results for two OMF designs are presented in Figure 6.

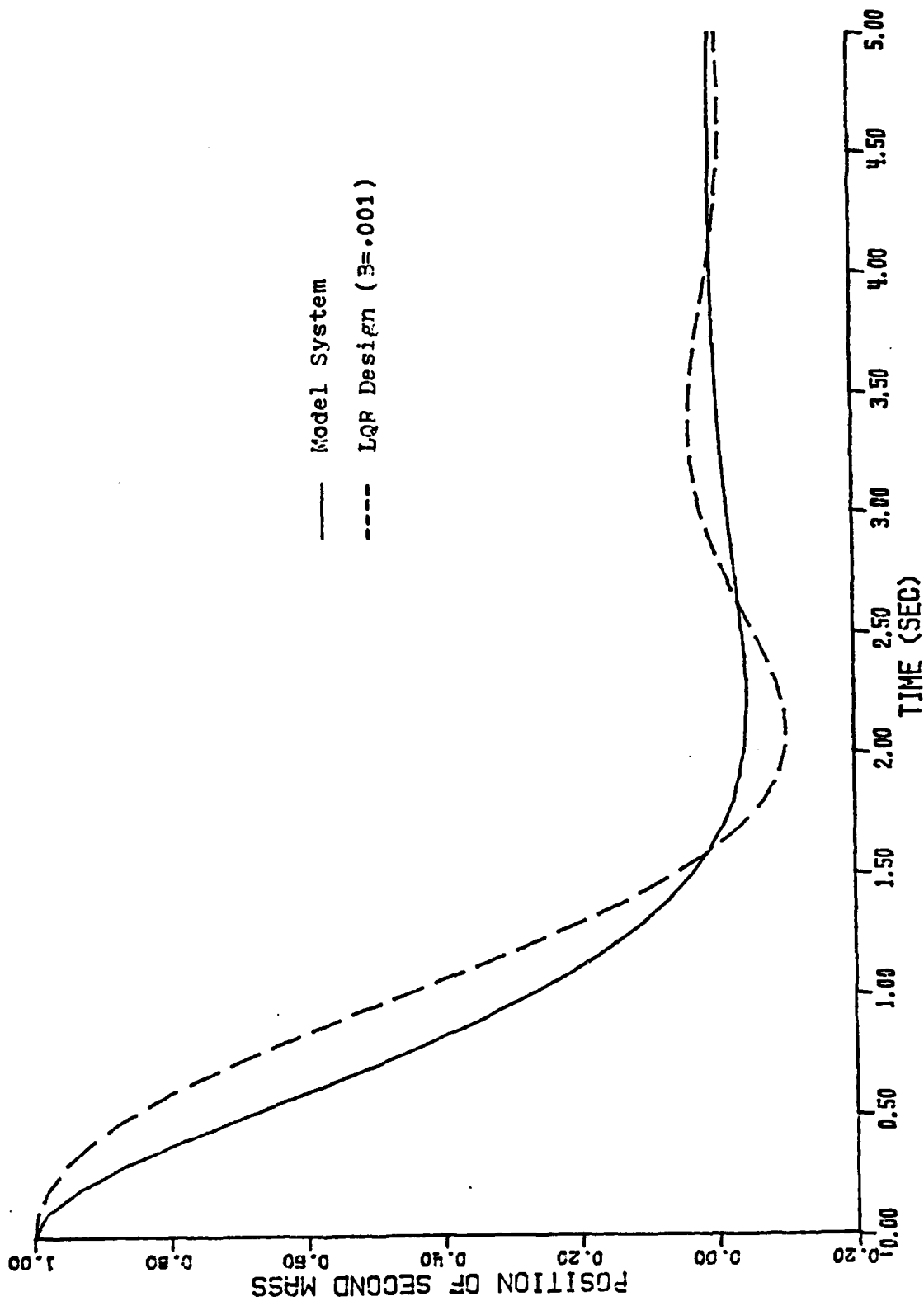


Figure 5: Impulse Response for LQR Design

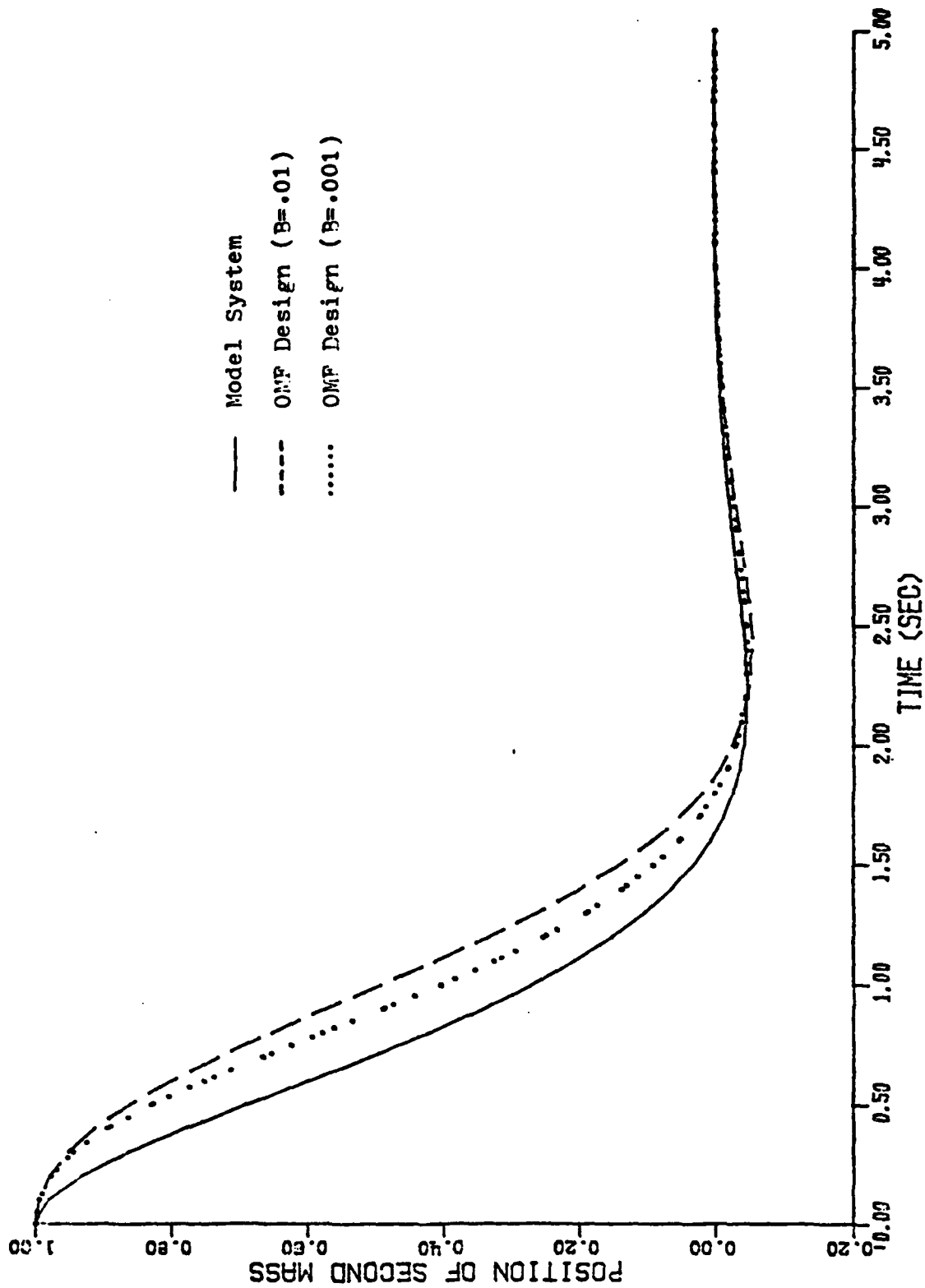


Figure 6: Impulse Response for OMF Designs

Chapter IV

MODEL FOLLOWING FOR INSENSITIVE CONTROL

4.1 REVIEW OF PAST EFFORTS

Unknown or varying parameters have always been a concern to control-system designers. Classically, this was handled by conservatively selecting the control-system gains and structure so that the predicted gain and phase margins, damping ratios, and time constants indicated that adequate performance could be achieved even for "worst case" conditions. This was then confirmed through simulation or system test, and if necessary, changes were made to the design.

On the surface, optimal control techniques appeared to make the design process much more objective. It was only necessary to input a few weighting parameters to the computer, and the "optimum" solution would be calculated. The key question of course, is "Optimum with respect to what criteria?" Kalman[14] has shown that almost any control law can be shown to be an optimum one, given the proper choice of cost function.

The sensitivity of a control system may be defined in many different ways, such as trajectory sensitivity, eigenvalue sensitivity, or cost function sensitivity. Typically though, the problem is that while control systems designed using standard linear quadratic regulator techniques may perform very well for the "nominal" conditions, with only small errors in the parameters, system performance can be significantly

degraded or even unstable. This has no doubt led to much confusion and embarrassment among newly graduated engineers, trying to apply the theory they learned in school to real-world problems. It has probably also helped to prevent more widespread use of modern control techniques in the aerospace industry. Actually, sensitivity is not an inherent characteristic of the optimal control approach, but rather a result of the designer failing to incorporate all of his design requirements into the criteria. As we shall see in a later example, the standard LQR theory can usually be used very successfully to give designs with as much stability as desired, even for large parameter uncertainties, as long as the designer takes these uncertainties into account during the design process. Although the resulting system no longer behaves "optimally" at the nominal parameter values, at least the instability problem is avoided.

Over the past few years, there have been a number of attempts to somehow automatically incorporate sensitivity characteristics into the design process. Palsson and Whitaker[23] and Peled[24] applied random-vector approaches with some success. Harvey and Pope[13] and Vinkler[30] produced excellent comparisons of a number of the most common methods for sensitivity reduction. Although some of the methods appear to be better than others, no single technique has been found to be clearly superior to the others in all cases. Almost all of the methods shared two common characteristics. First, computational requirements were very great. Most of the algorithms required the use of gradient-search routines to find the minimum of a cost function, and many iterations were sometimes required. Second, and most importantly, system

performance and/or control use at the nominal parameter values was degraded, with the amount depending on how insensitive the system was asked to be. In addition, although stability was guaranteed in some of the methods, system eigenvalues could vary dramatically with changes in the parameters. The model-following method discussed later in this chapter is shown to avoid these undesirable characteristics.

4.2 FEEDFORWARD/FEEDBACK EQUIVALENCE

Suppose we have a plant

$$\dot{x} = Fx + Gu,$$

and a model, of the same dimension,

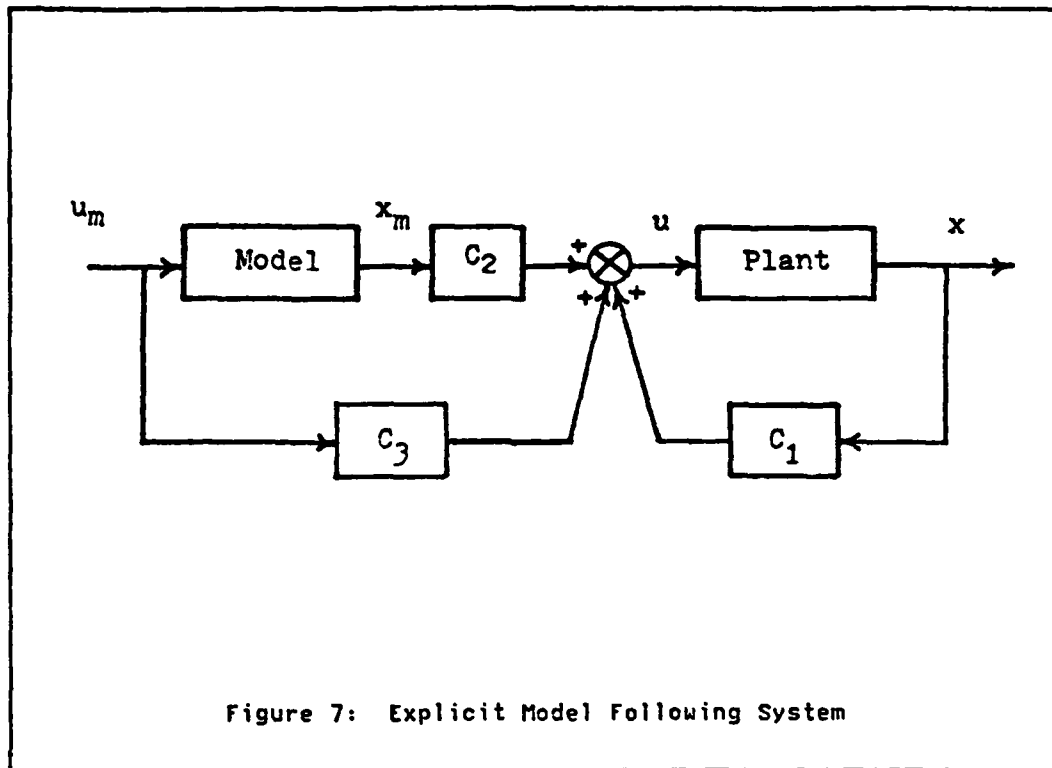
$$\dot{x}_m = F_m x_m + G_m u_m.$$

Let us implement an explicit model-following control system, with control law of the form

$$u = C_1 x + C_2 x_m + C_3 u_m.$$

The system structure is then as shown in Figure 7.

By adding and subtracting $C_2 x$, the control law may be rewritten as



$$u = C_1 x + C_2 x_m + (C_2 x - C_2 x) + C_3 u_m.$$

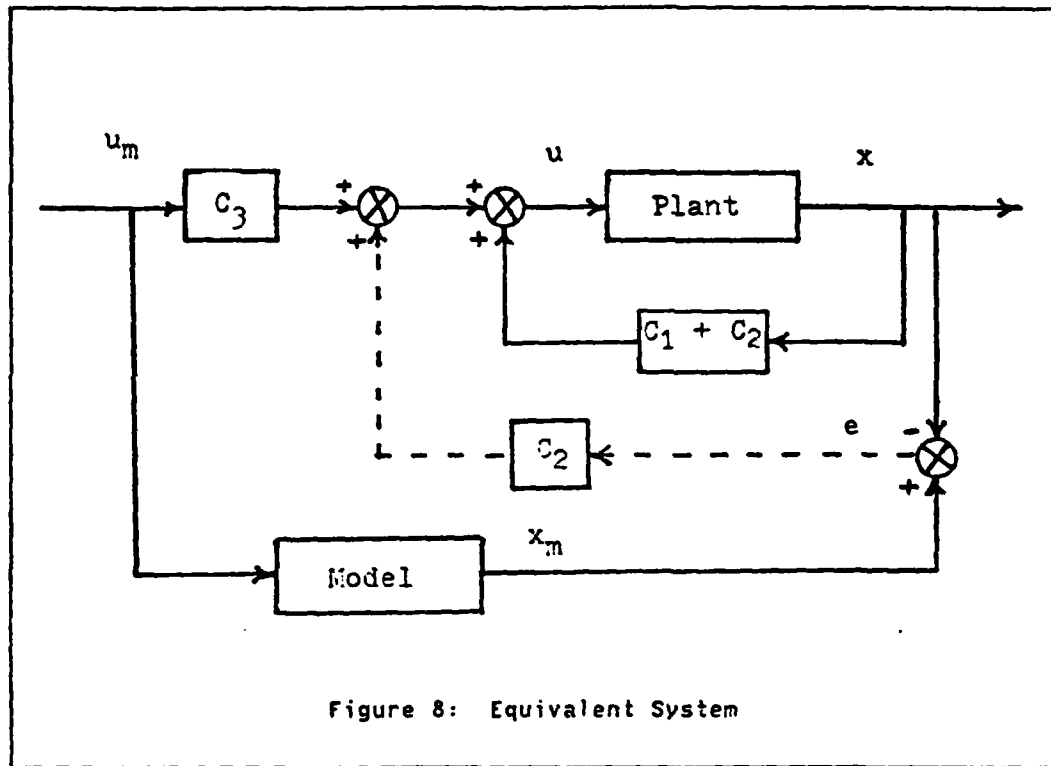
Thus,

$$u = (C_1 + C_2)x + C_2(x_m - x) + C_3 u_m.$$

Defining $e = x_m - x$, we obtain

$$u = (C_1 + C_2)x + C_2 e + C_3 u_m.$$

The block diagram in Figure 8 can thus be seen to be equivalent to the one in Figure 7.



Suppose the model and the closed-loop plant are exactly the same, and the initial conditions are also identical. Then x and x_m will always be equal, and the error, e , will always be zero. In this case, as Figure 8 makes clear, it is $(C_1 + C_2)$ which has determined the closed-loop plant dynamics. The match of plant and model states could therefore have been achieved with only feedback from the plant states ($C_2 = 0$), with only feedforward from the model states ($C_1 = 0$), or with any combination of the two, as long as the sum of C_1 and C_2 remained the same.

As the above discussion indicates, Implicit Model Following is just as effective as Explicit Model Following in terms of matching dynamics (assuming plant and model are of the same dimension and all of the states are matched). Thus, in cases where the plant parameters are well known and there are no unknown disturbances, or where a conservative design is acceptable, there is no need to implement the model. Where this is not the case, the explicit formulation should be used, as discussed in the following sections.

4.3 DESCRIPTION OF THE DESIGN METHOD

Many authors have stated their "intuitive" feelings that explicit model-following systems should have some advantages in terms of system sensitivity, at least as compared to Implicit Model Following systems. It certainly seems reasonable that the ability to generate an error signal between the plant and a model system and using it as part of the control input would help to keep plant states "close" to the model states. However, unless the system roots are selected carefully, the transient response of the plant may bear no resemblance to the model response.

Tyler[29] and others formulated the explicit model-following problem as finding the control which minimized the cost function

$$J = \int_0^{\infty} (e^t A e + u^t B u) dt.$$

Thus, they reasoned that an optimal tradeoff could be accomplished between the error and the control effort used.

Actually, the above cost function was usually computed using

$$J = \int_0^{\infty} [(x_m - x)^t A (x_m - x) + u^t B u] dt.$$

As a result, optimal gains were computed for both x and x_m , even though, as we saw in the preceeding section, the model-following ability of a system is not dependent on whether it is implemented with feedforward, feedback, or a combination of the two. In other words, the gains which minimize the above cost function do not uniquely determine the plant response, and different combinations of feedforward and feedback gains will have substantially different effects for off-nominal conditions.

Kriendler[16] noted the poor dynamics-matching abilities of this type of system, which caused him to prefer Implicit Model Following. The poor matches really had nothing to do with whether the system was explicit or implicit, but rather was a function of using state matching instead of dynamics matching. Kriendler also acknowledged that Explicit Model Following might have better sensitivity properties, although, since he used Tyler's method (state matching) to calculate the gains, he found no significant difference in sensitivity between the two methods. In any event, to incorporate the advantages of both explicit and implicit model-following systems, he proposed (although he did not investigate) the cost function

$$J = \int_0^{\infty} [(\dot{y} - F_m y)^t A_1 (\dot{y} - F_m y) + (y_m - y)^t A_2 (y_m - y) + u^t B u] dt$$

This does appear to represent a great improvement over the standard state-matching formulation. The error-rate term tends to match the dynamics, while the error term tends to speed up the plant states by increasing the feedback gains. This increased feedback tends to desensitize the system to parameter errors. The disadvantage is that changes in A_2 to adjust system sensitivity will also result in changes to the nominal plant dynamics/transient response. Instead, it is claimed that the design process should be accomplished twice: once to establish the plant dynamics at the nominal parameter values, and once to establish the error dynamics (and therefore the system sensitivity). By doing the design in this manner, system performance at the nominal parameter values becomes independent of the sensitivity.

The proposed design method, which we call Model Following for Insensitive Control, is then as follows:

1. Perform a design, using any method (root locus, model following, optimal control, etc.), to select the nominal plant response. These gains become $C_4 = C_1 + C_2$ and C_3 .
2. Implement a model of this closed-loop system.
3. Do another design to select the error roots, which should be chosen faster than the plant roots. These gains become C_1 .

4. Calculate $C_2 = C_4 - C_1$.

5. Implement the control law as $u = C_1x + C_2x_m + C_3u_m$.

4.4 THEORETICAL DEVELOPMENT

Let the behavior of a plant with nominal parameter values be given by

$$\dot{x} = Fx + Gu.$$

However, due to parameter uncertainty and/or slowly varying parameters, suppose the plant actually responds according to

$$\dot{x} = (F + \Delta F)x + Gu.$$

Through the use of model-following techniques, optimal control, root locus, or any other method, assume that a satisfactory closed-loop control law can be obtained for nominal values of the parameters. Let this control law be given by

$$u = C_1x + C_3u_m.$$

Then, for nominal values of the parameters, we have

$$\dot{x} = Fx + G(C_1x + C_3u_m)$$

$$= (F + GC_t)x + GC_3u_m.$$

Build a math model of this system, i.e.,

$$\dot{x}_m = (F + GC_t)x_m + GC_3u_m$$

$$= F_m x_m + G_m u_m.$$

For the second design, we again may use whatever method we desire, but the objective this time is to select the error roots. For an insensitive design, these roots should be chosen faster than the nominal-plant roots. The resulting gains are designated C_1 .

We then use a control law

$$u = C_1x + C_2x_m + C_3u_m$$

$$= C_t x + C_2 e + C_3 u_m,$$

where $e = x_m - x$ and $C_t = C_1 + C_2$.

This results in

$$\dot{x} = (F + \Delta F)x + G(C_t x + C_2 e + C_3 u_m)$$

$$= (F + \Delta F + GC_t)x + GC_2 e + GC_3 u_m.$$

Subtracting, we get

$$\begin{aligned}\dot{e} &= (F + GC_t)e - \Delta Fx - GC_2e \\ &= (F + GC_t - GC_2)e - \Delta Fx.\end{aligned}$$

The system is thus described by

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} F + \Delta F + GC_t & GC_2 \\ -\Delta F & F + GC_t - GC_2 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} GC_3 \\ 0 \end{bmatrix} u_m,$$

with a characteristic equation given by

$$\begin{vmatrix} sI - F - \Delta F - GC_t & -GC_2 \\ \Delta F & sI - F - GC_t + GC_2 \end{vmatrix} = 0.$$

Since the determinant of a matrix is not changed by elementary row and column operations, we can write

$$\begin{vmatrix} sI - F - GC_t & sI - F - GC_t \\ \Delta F & sI - F - GC_t + GC_2 \end{vmatrix} = 0$$

and

$$\begin{vmatrix} sI - F - GC_1 & 0 \\ \Delta F & sI - F - \Delta F - GC_1 + GC_2 \end{vmatrix} = 0.$$

This can be written as

$$(sI - F - GC_1)(sI - F - \Delta F - GC_1 + GC_2) = 0,$$

or equivalently,

$$(sI - F_m)(sI - F - \Delta F - GC_1) = 0.$$

Thus, given an appropriate choice for C_1 , the roots of $(sI - F - \Delta F - GC_1)$ can usually be made fast enough to keep the roots of $(sI - F_m)$ as the dominant system roots, even for arbitrarily large ΔF . If the error roots are made sufficiently fast, they will not significantly affect the system response, regardless of the value of ΔF . Also, if the plant and model start with identical initial conditions and $\Delta F = 0$, the error roots will never even be excited, since the system equations will be

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} F + GC_1 & GC_2 \\ 0 & F + GC_1 - GC_2 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} GC_3 \\ 0 \end{bmatrix} u_m.$$

4.5 EXAMPLE: FIRST-ORDER SYSTEM

In order to get a better understanding of how Model Following for Insensitive Control works, let us consider the simple, first-order system used by Shenkar[26]. The plant is described by

$$\dot{x} = -(1 + \Delta F)x + u,$$

where ΔF represents the uncertainty in a plant parameter. The value of ΔF may be anything from -1 to 1, with 0 being the nominal value. Three different design techniques were evaluated: the standard Linear Quadratic Regulator (LQR) technique, the Average Cost Function (ACF) approach (representative of the methods of Vinkler, Ashkenazi, and Shenkar), and Model Following for Insensitive Control (MFIC).

For the LQR design, we assume that our design requirements are met by letting $A=B=1$ for the nominal plant. However, as might be expected, the system dynamics are very dependent on the value of the uncertain parameter. The impulse responses for the nominal and extreme values of ΔF are shown in Figure 9.

With the Average Cost Function design there is definitely some improvement, but we still have a wide range of responses for the different values of ΔF . Also, even if $\Delta F=0$, the response is now different from the nominal LQR design. Results for this design are shown in Figure 10.

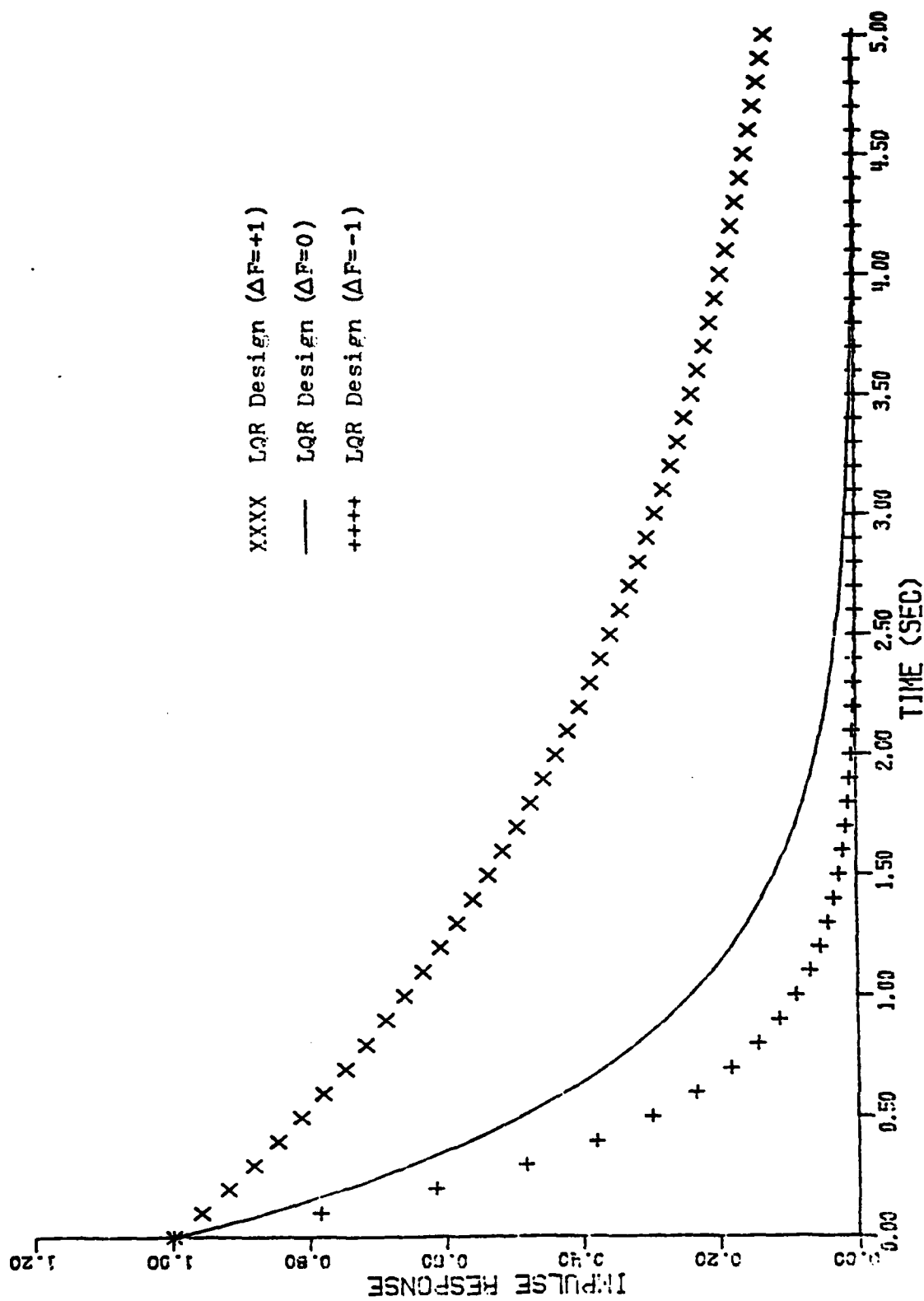


Figure 9: First-Order System - LQR Design

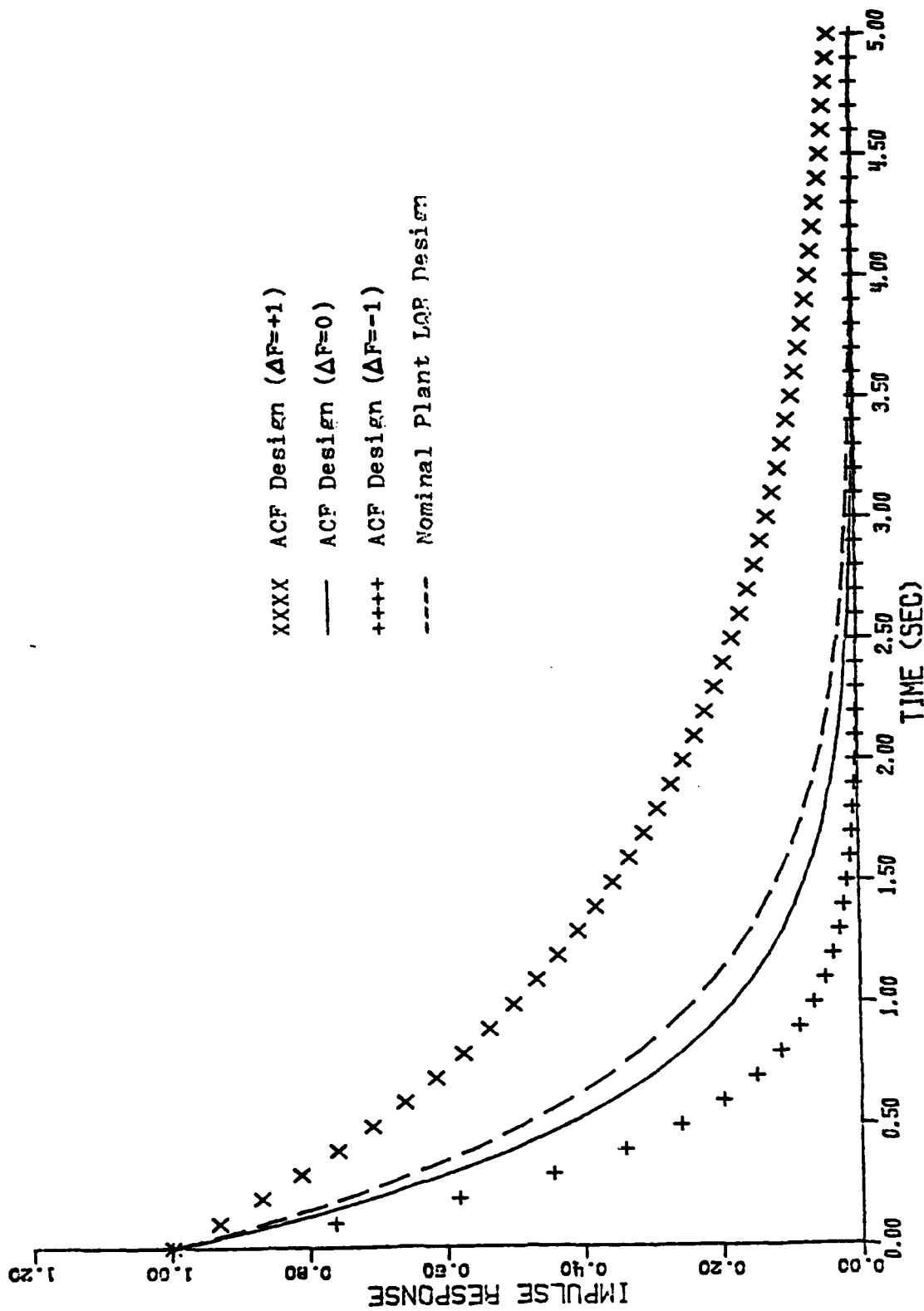


Figure 10: First-Order System - ACF Design

For the Model Following for Insensitive Control design, the closed-loop root for the nominal plant was chosen to be the same as the nominal LQR system, while the error root was arbitrarily chosen to be at $s=-10$. With this system, the impulse responses are almost identical, regardless of the value of ΔF . The responses are shown in Figure 11.

If a more insensitive design is required, one can simply select a faster error root. Without disturbances, the control levels required are directly related to the transient response. Thus, the control magnitudes for the MFIC design are essentially the same as the control magnitudes which would have been required for a design in which ΔF was known in advance. Control bandwidth however, obviously must be increased as the error roots are speeded up (which may be costly).

To understand what is happening physically, it is helpful to think in terms of the system transfer functions. The open-loop plant is described by $1/(s+1)$. The desired closed-loop system, which is attained with the nominal LQR design, has a transfer function of $1/(s+1.414)$. The ACF design has a slightly higher feedback gain, resulting in a transfer function of $1/(s+1.689)$. With the MFIC design, the plant is greatly speeded up, in this case to $1/(s+10)$. However, the feedforward from the model and the model input form a prefilter or "command conditioner" with a transfer function of $(s+10)/(s+1.414)$. Thus, with nominal parameters, the speeded-up plant root is exactly cancelled, resulting in the desired system transfer function. With a non-zero ΔF , perfect cancellation is not attained, but because the pole and zero are close to each other and both are much faster than the rest of the system, they do not significantly affect the response.

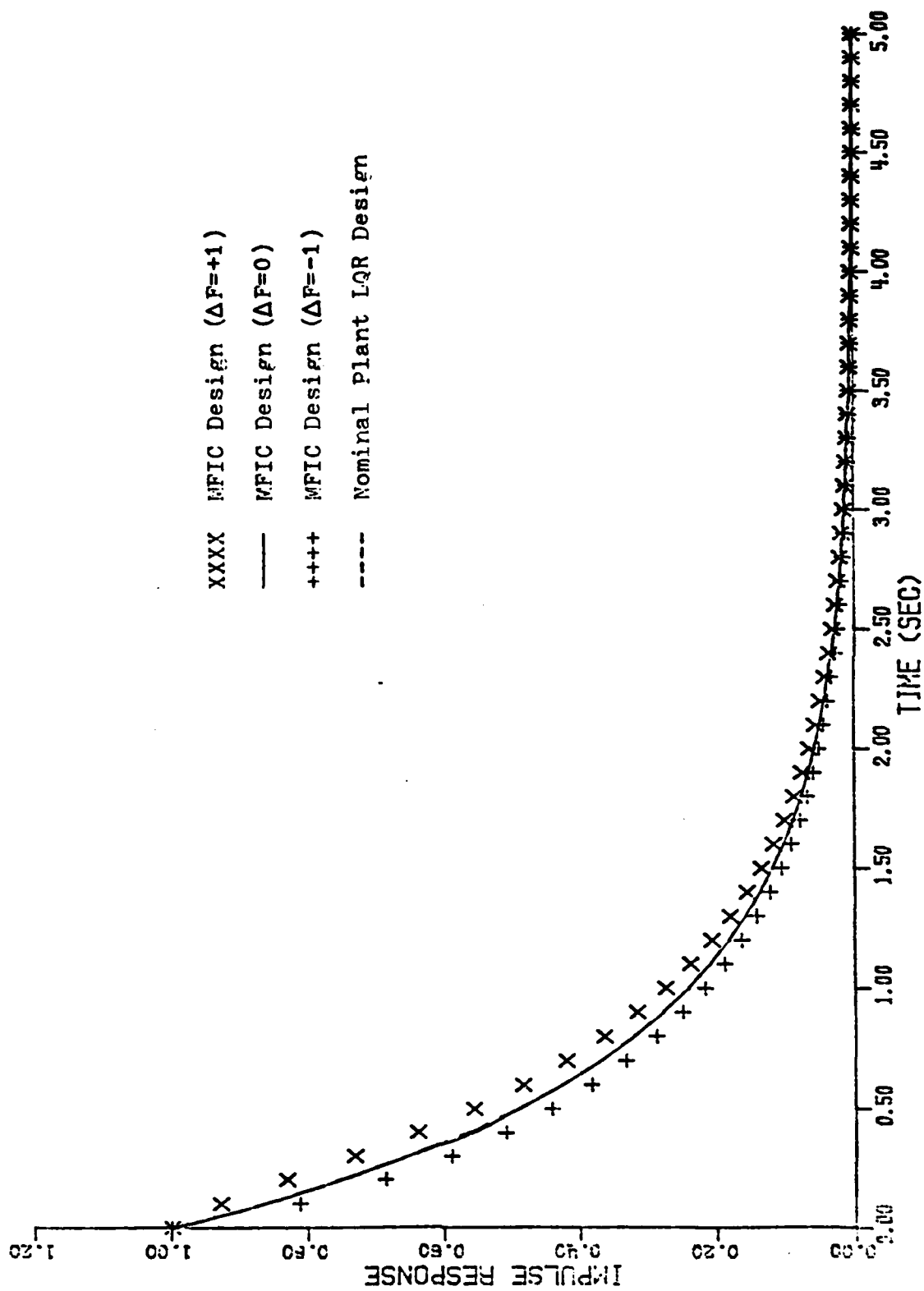


Figure 11: First-Order System - MFIC Design

The beneficial effects of this design method stem from two factors: first, the increased feedback gains, which have a desensitizing effect well-known to classical designers, and second, the cancellation of the speeded-up plant roots. Note that either of these items by itself would not yield the desired results. Just increasing the feedback gains would lower the system sensitivity, but the system response would be overdamped or faster than desired. On the other hand, just cancelling the plant roots would allow us to achieve the desired closed-loop roots, but it does nothing for the sensitivity problem.

The MFIC design procedure is primarily oriented to obtaining a desirable transient response, rather than to disturbance rejection. On the other hand, disturbance rejection is a very important part of the Average Cost Function approaches. To try to compare the two methods from a different perspective, the same first-order example was used, but with the addition of a process-noise source, with a power-spectral-density and a process-noise distribution element of 1. The two components of the cost function - state excursions and control use - were then plotted against feedback gain in Figures 12 and 13.

As might be expected, the greater the negative feedback gain, the tighter the control system is able to regulate the state, and hence the smaller the state excursions. Likewise, the greater the feedback gains are, the higher the control level that is used to handle the disturbance. By adding these two terms together, the total value of the cost function is obtained. Plots of this total cost function versus the uncertain parameter are presented for the LQR design, the ACF design, and two MFIC designs in Figure 14.

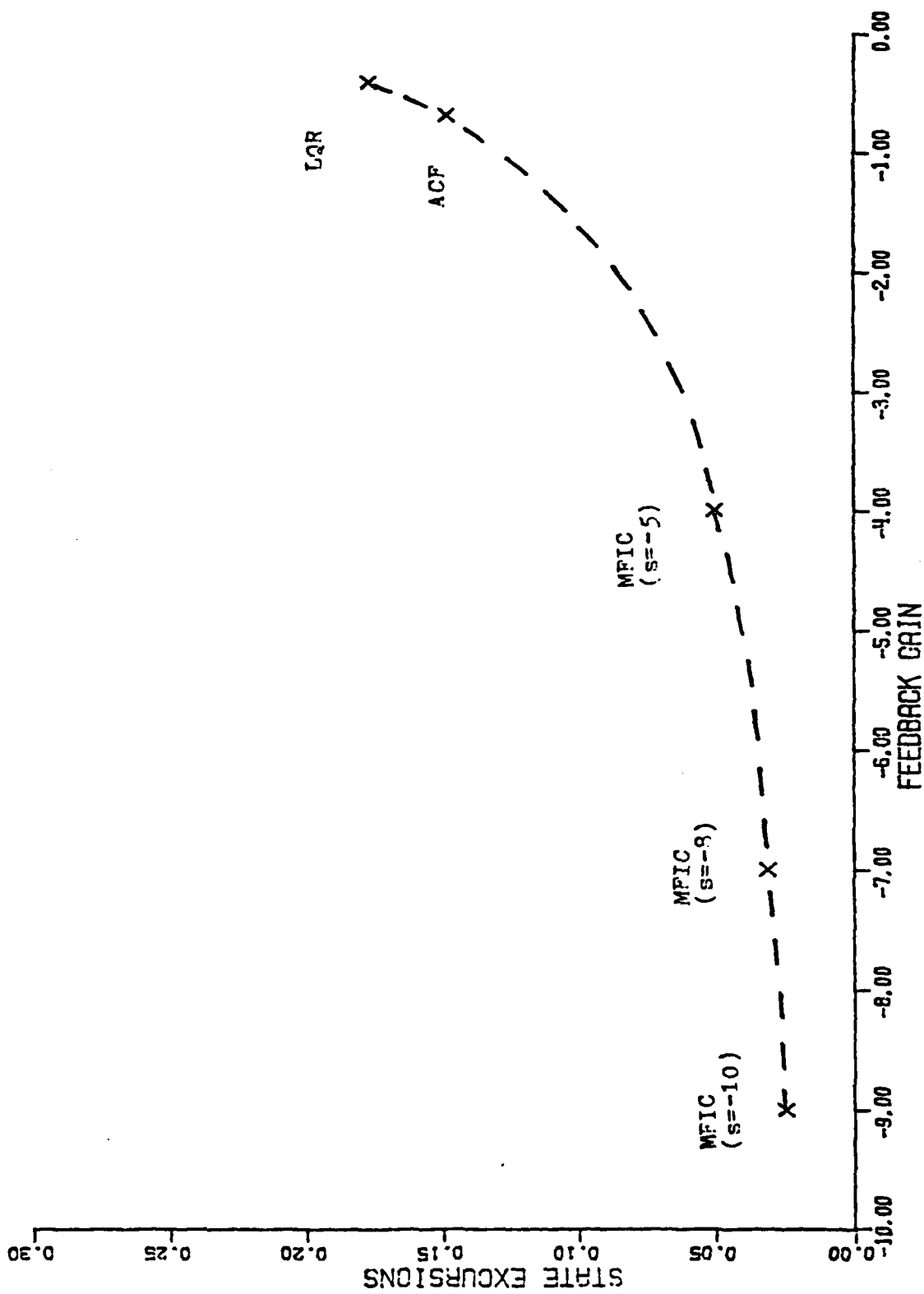


Figure 12: State Excursions for First-Order System with Process Noise

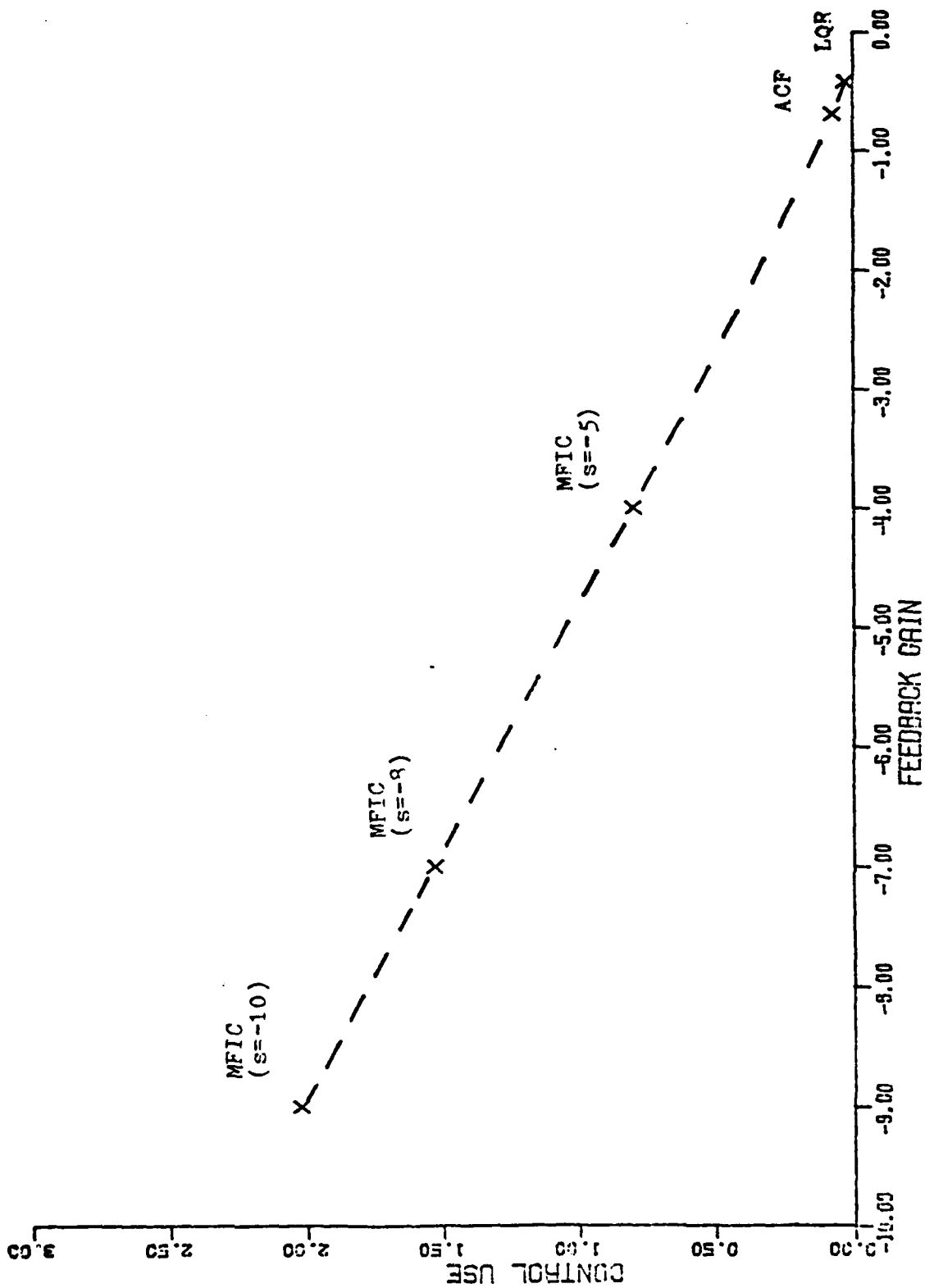


Figure 13: Control Use for First-Order System with Process Noise

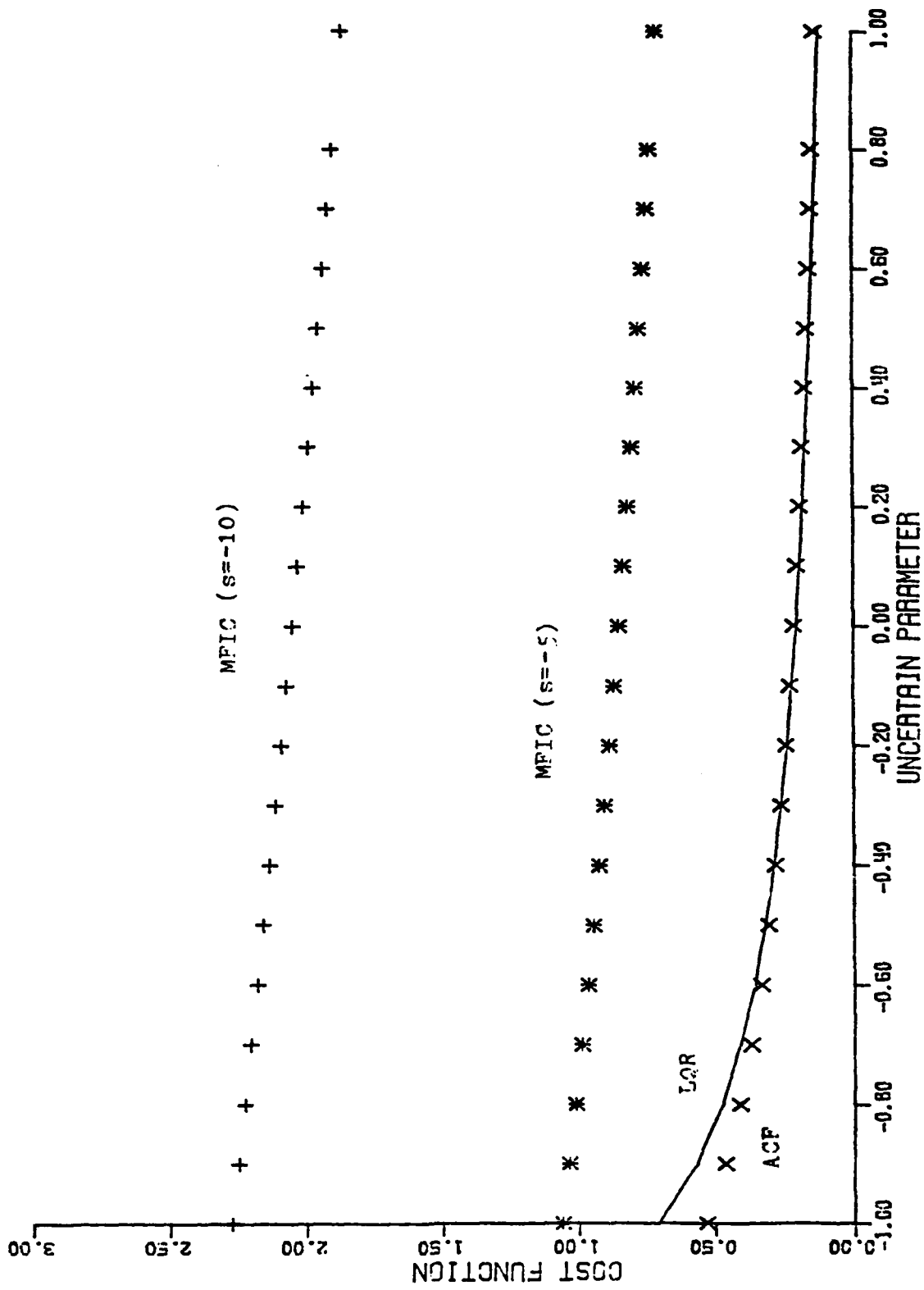


Figure 14: Cost Function Comparison

The advantage of the ACF design is supposed to be that by accepting a slightly higher value of the cost function for positive and zero values of the uncertain parameter, a substantial improvement can be achieved for negative values of the parameter. However, the MFIC design, which had much better transient responses, shows significantly higher values of the cost function (which contains control effort) for any ΔF . In fact, the faster the error root is made, (and hence the more insensitive the resulting system), the higher the cost function becomes.

The message from this example is that it is important for the designer to be sure that he has chosen the cost function which properly represents the problem he is trying to solve. In this case, by using a model-following structure and increased feedback gain, it was possible to achieve a significant improvement in transient response compared to the so-called "optimal" design. The cost of this improvement was the increased control usage due to the process noise, which may or may not be important to the designer.

4.6 EXAMPLE: SECOND-ORDER SYSTEM

For our next example, consider the second-order system studied by Vinkler[30]:

$$\dot{x} = Fx + Gu,$$

with

$$F = \begin{bmatrix} 0 & 1 \\ -2+\omega_1 & 1+\omega_2 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and

$$-1 \leq \omega_1 \leq 1$$

$$-1 \leq \omega_2 \leq 1.5.$$

With weighting matrices of

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and } B = 10,$$

a standard LQR design yields an unstable system for some values of ω .

Vinkler evaluated several design techniques which have been suggested for handling uncertain parameters. He then described two methods for obtaining better results: the Multistep Guaranteed Cost Control method (MGCC) and the Modified Discrete Expected Cost method (MDEC). Both methods require substantial computational effort and special computer programs during the design stage. Actually, in many problems the designer can obtain satisfactory results with standard LQR techniques, as

long as they are used properly. If the designer is aware that the system has uncertain parameters, he should make his nominal design more stable than he ordinarily would. Then, even in the presence of destabilizing values of the parameters, an adequate degree of damping is still obtained. For example, in the present problem, if we choose

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 50 \end{bmatrix} \text{ and } B = 10,$$

the closed-loop response is stable for all allowable values of ω . The disadvantage with this approach (as well as with Vinkler's) is that in order to get a satisfactory stability level with "unfavorable" values of the uncertain parameter, we have to accept overdamped or faster than desired responses for nominal or "favorable" values of the uncertain parameter. By using Model Following for Insensitive Control, we can avoid this result. For this example, the nominal-plant roots were chosen to be the same as for the original LQR design. The error roots were arbitrarily chosen to both be at $s=-5$. The resulting eigenvalues for each method are shown in Table 1.

Notice that with the MFIC design the dominant eigenvalues remain constant, and in fact, are identical to the desired nominal LQR roots. If a more insensitive design is desired, faster error roots can be selected (with correspondingly increased control effort), so that their effect on the system response becomes negligibly small.

TABLE 1
Second-Order System - Eigenvalue Comparison

	ΔF	$F + \Delta F$	eigenvalues
LQR	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ -1 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$
MGCC	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ -1 & 1.5 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 1 & 1.5 \end{bmatrix}$
PDRC	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ -1 & 1.5 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 1 & 1.5 \end{bmatrix}$
Better LQR	$\begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -3 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 2.5 \end{bmatrix}$
MFIC	$\begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -3 & 2.5 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 2.5 \end{bmatrix}$

4.7 EXAMPLE: SIMULATION OF A SUPERSONIC TRANSPORT

As a final example, we apply the MFIC design method to the problem of simulating the longitudinal motions of a supersonic transport. This problem has appeared at least twice in the technical literature: Winsor and Roy[31] used it to demonstrate a trajectory sensitivity method, and Landau and Courtiol[18] gave a solution for an adaptive-control approach. The plant aircraft is the Total In-Flight Simulator (TIFS), a modified Convair C-131B which has side-force generators and direct-lift flaps in addition to the normal control surfaces.

The plant equations of motion are given by

$$\dot{x} = Fx + Gu,$$

with

$$x^t = [\theta \quad q \quad \alpha \quad v]$$

$$F = \begin{bmatrix} 0. & 1.0 & 0. & 0. \\ 1.401 \times 10^{-4} & M_q & -1.9513 & .0133 \\ -2.505 \times 10^{-4} & 1.0 & -1.3239 & -.0238 \\ -.5610 & 0. & .3580 & -.0279 \end{bmatrix}$$

$$G = \begin{bmatrix} 0. & 0. & 0. \\ -5.3307 & 6.447 \times 10^{-3} & -.2669 \\ -.1600 & -1.155 \times 10^{-2} & -.2511 \\ 0. & .1060 & .0862 \end{bmatrix}$$

and

$$u^t = [6e \ 6t \ 6f].$$

The state vector consists of pitch angle, pitch rate, angle of attack, and velocity. The control vector is made up of elevator deflection, throttle position, and flap deflection.

The model equations are

$$\dot{x}_m = F_m x_m + G_m u_m,$$

with

$$x_m = [\theta_m \ q_m \ \alpha_m \ v_m]$$

$$F_m = \begin{bmatrix} 0. & 1.0 & 0. & 0. \\ 5.318 \times 10^{-7} & -.4179 & -.1202 & 2.319 \times 10^{-3} \\ -4.619 \times 10^{-9} & 1.0 & -.7523 & -2.387 \times 10^{-2} \\ -.5614 & 0. & .3002 & -1.743 \times 10^{-2} \end{bmatrix}$$

$$G_m = \begin{bmatrix} 0. & 0. \\ -.1717 & 7.451 \times 10^{-6} \\ -.0238 & -7.783 \times 10^{-5} \\ 0. & 3.685 \times 10^{-3} \end{bmatrix}$$

and

$$u_m^t = [6e_m \quad 6t_m].$$

The parameter M_q has a nominal value of -2.038, but it is assumed to vary from -.558 to -3.558, which represents a variation of approximately 75% around the nominal value. Since perfect model following is possible for this system, the plant will exactly match the model responses for nominal M_q . However, with $M_q = -3.558$, the responses change significantly. By using the MFIC design method, we can obtain much better results. Time histories for a step elevator input for the model, the off-nominal plant, and the off-nominal plant using a MFIC design are given in Figures 15-17. The error roots were arbitrarily chosen to be at $s = -10, -10, -2$, and -2 , with the faster roots corresponding to the pitch-angle and pitch-rate errors. As shown in Figure 18, the maximum pitch-angle error was less than .005 degrees. The angle-of-attack and velocity responses had zero error. This represents a considerable improvement over the trajectory-sensitivity method tried by Winsor and Roy and the adaptive design given by Landau and Courtiol. Control uses are reasonable and are presented in Figures 19-21.

The system response to an initial error between the plant and the model pitch angle was also investigated, with nominal values for the parameters. The results are shown in Figure 22. The error is driven rapidly to zero, and unlike the previously published efforts, errors are not excited in the angle of attack and velocity states.

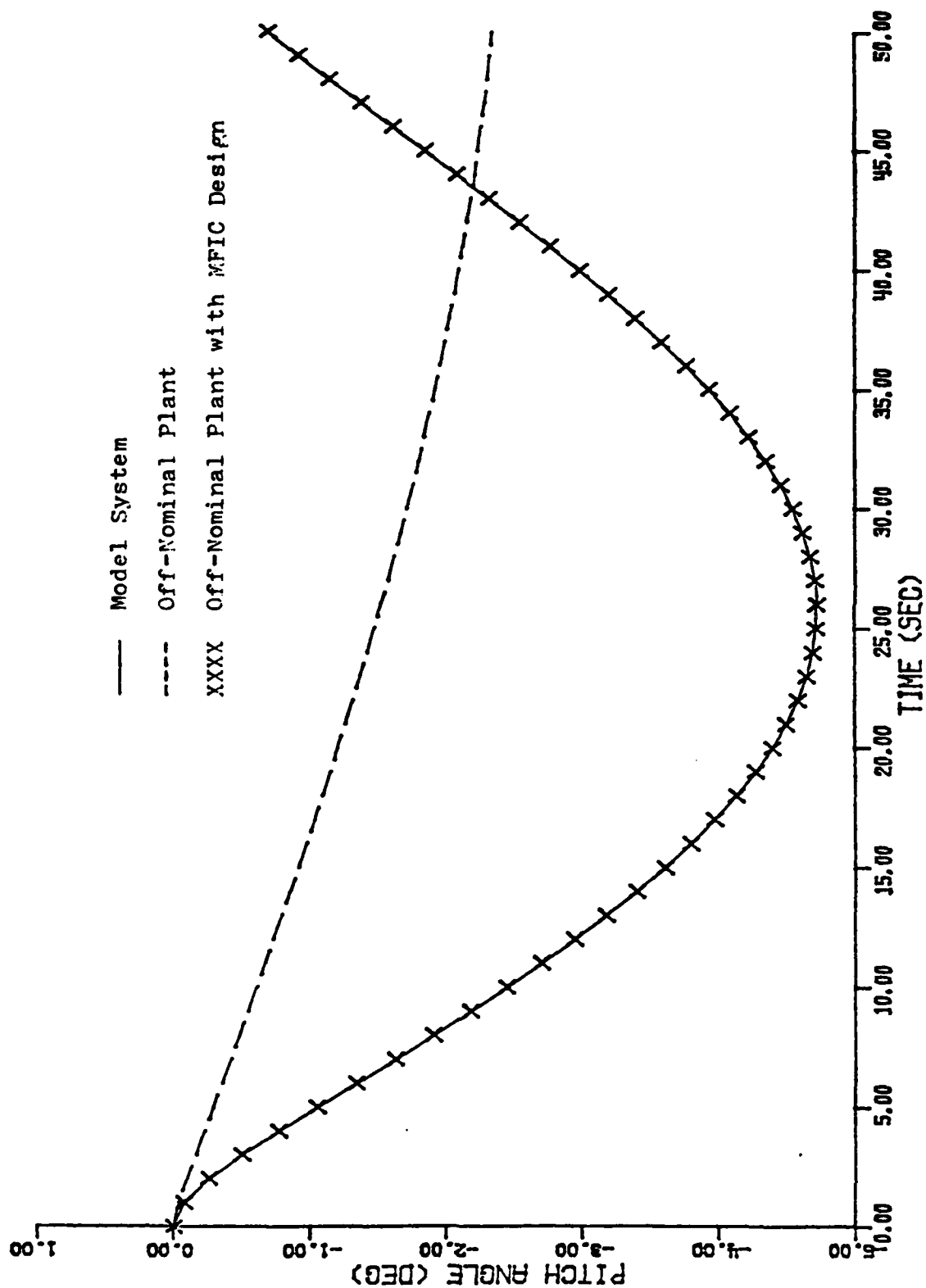


Figure 15: C-131/SST Pitch Angle Response

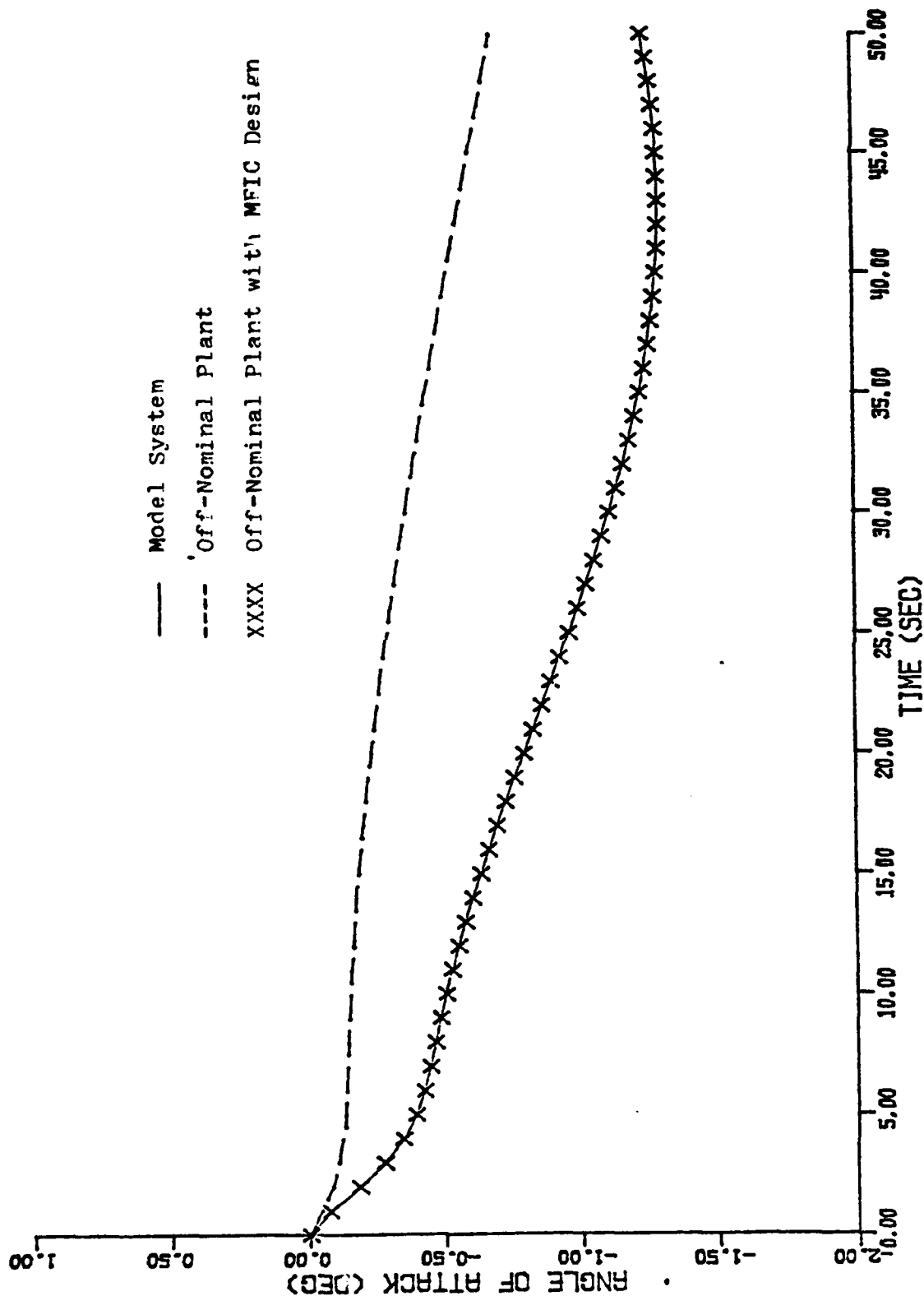


Figure 16: C-131/SSR Angle-of-Attack Response

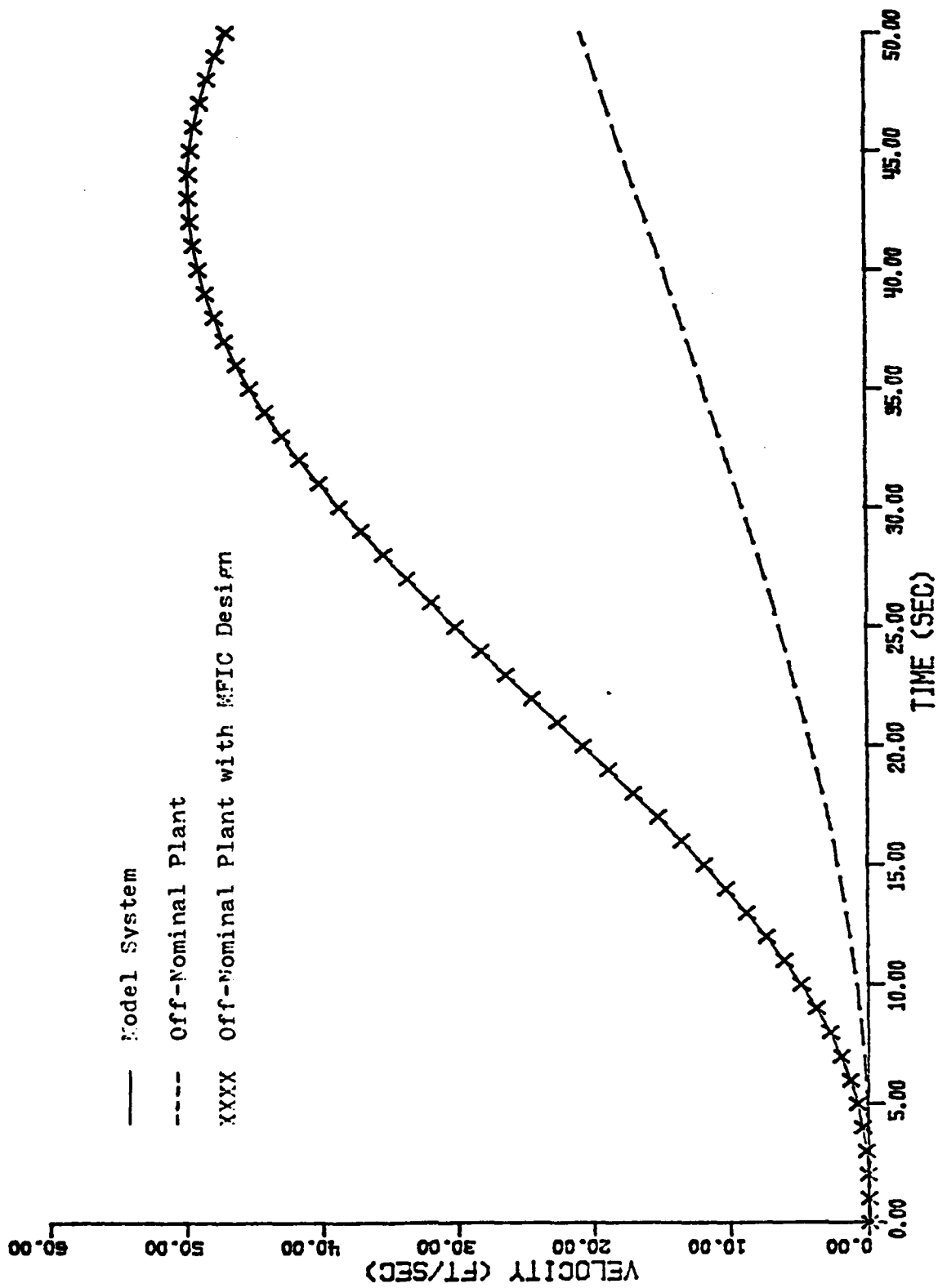


Figure 17: C-131/SST Velocity Response

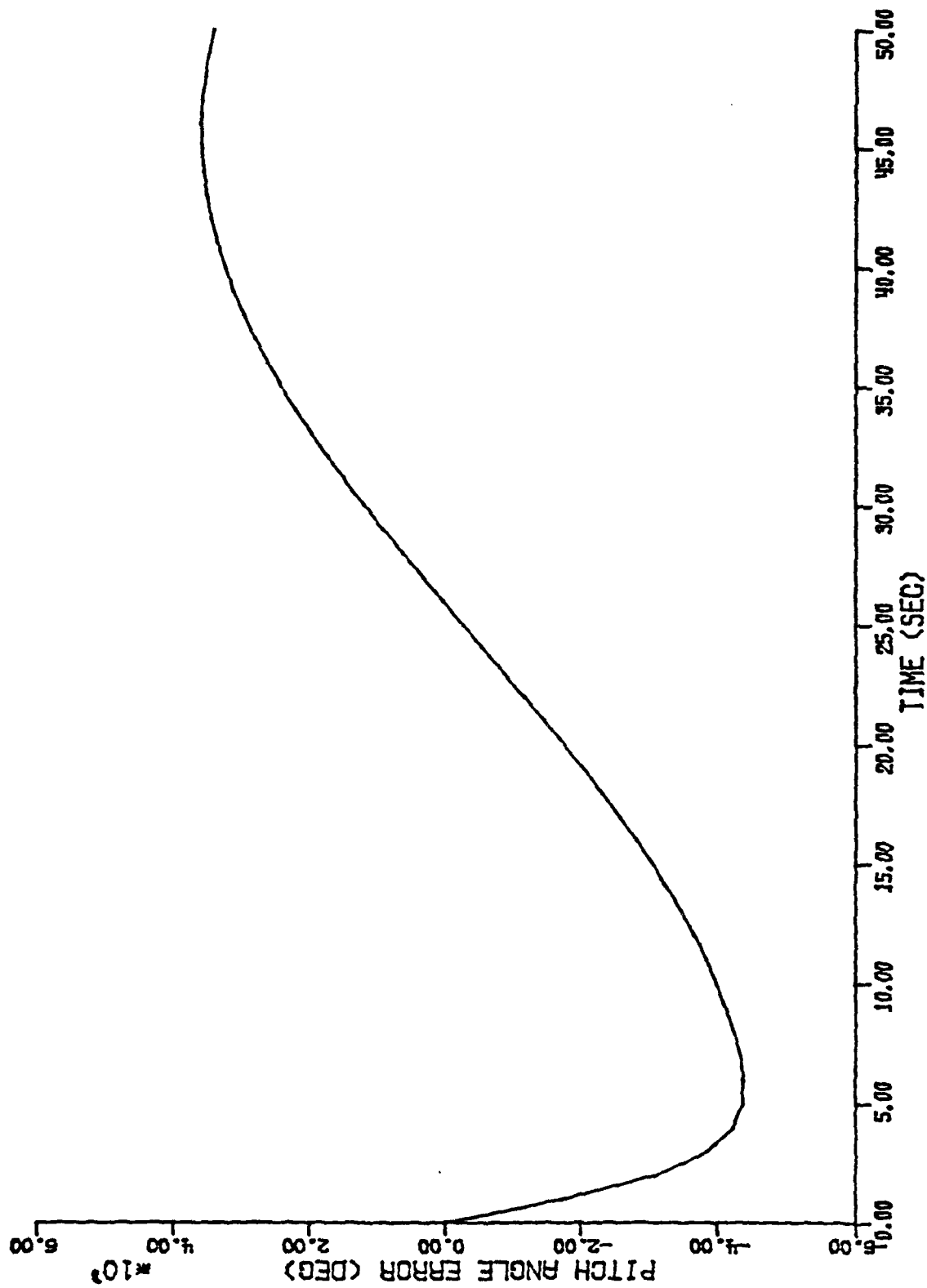


Figure 18: C-131/SS7 Pitch Angle Error with MFIC

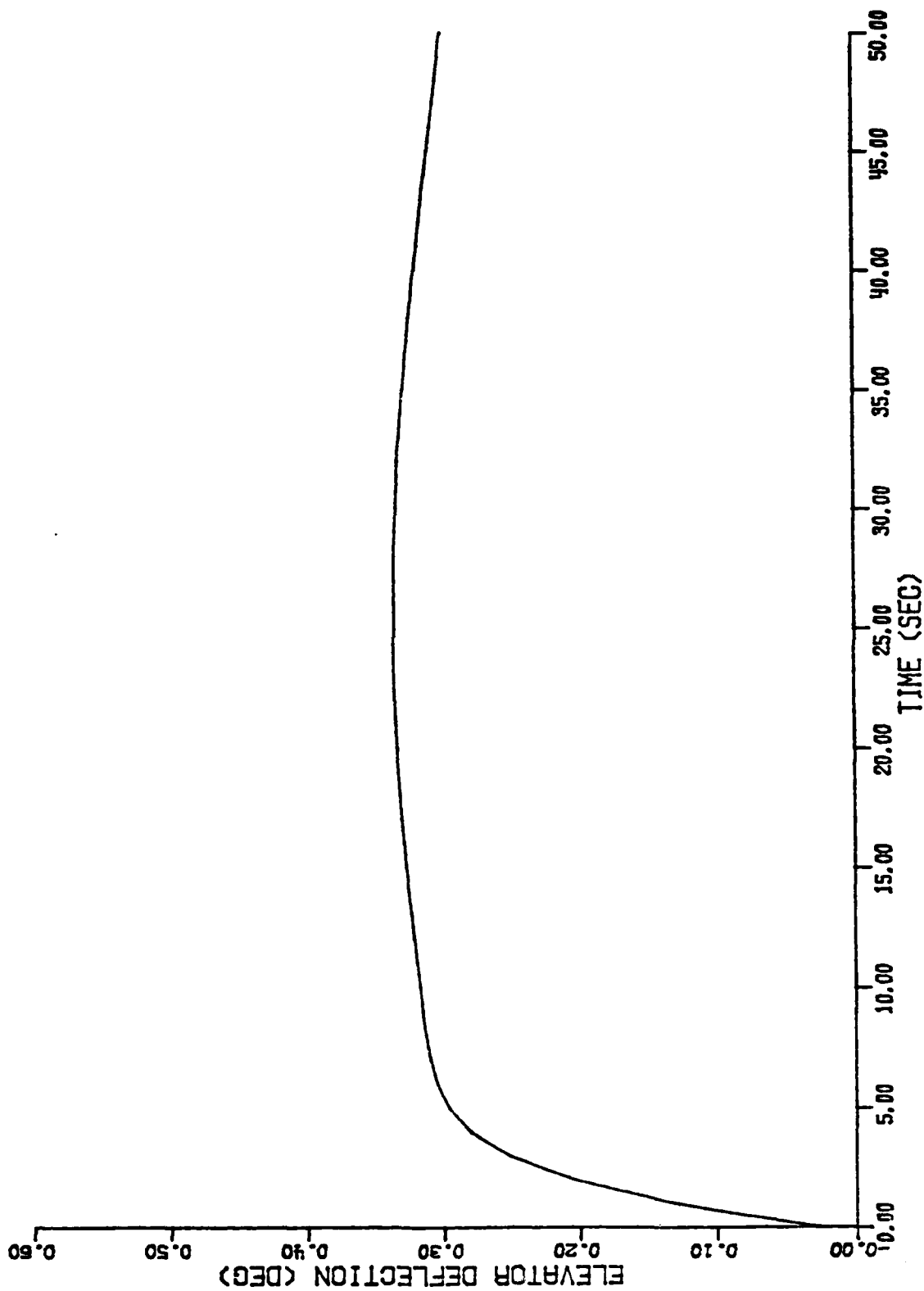


Figure 19: C-131/SST Elevator Deflection with MFIC

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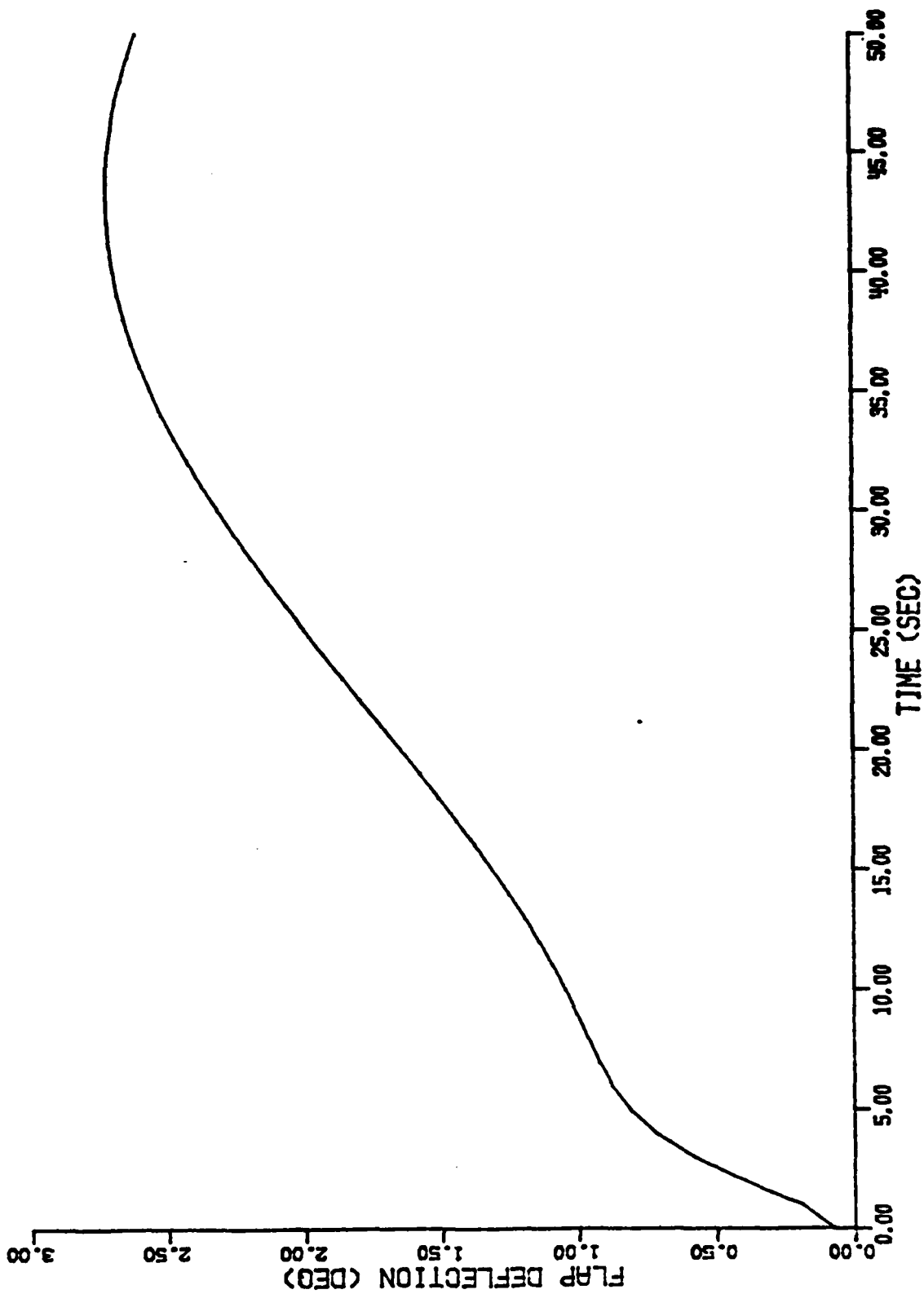


Figure 20: C-131/SST Flap Deflection with MFIC

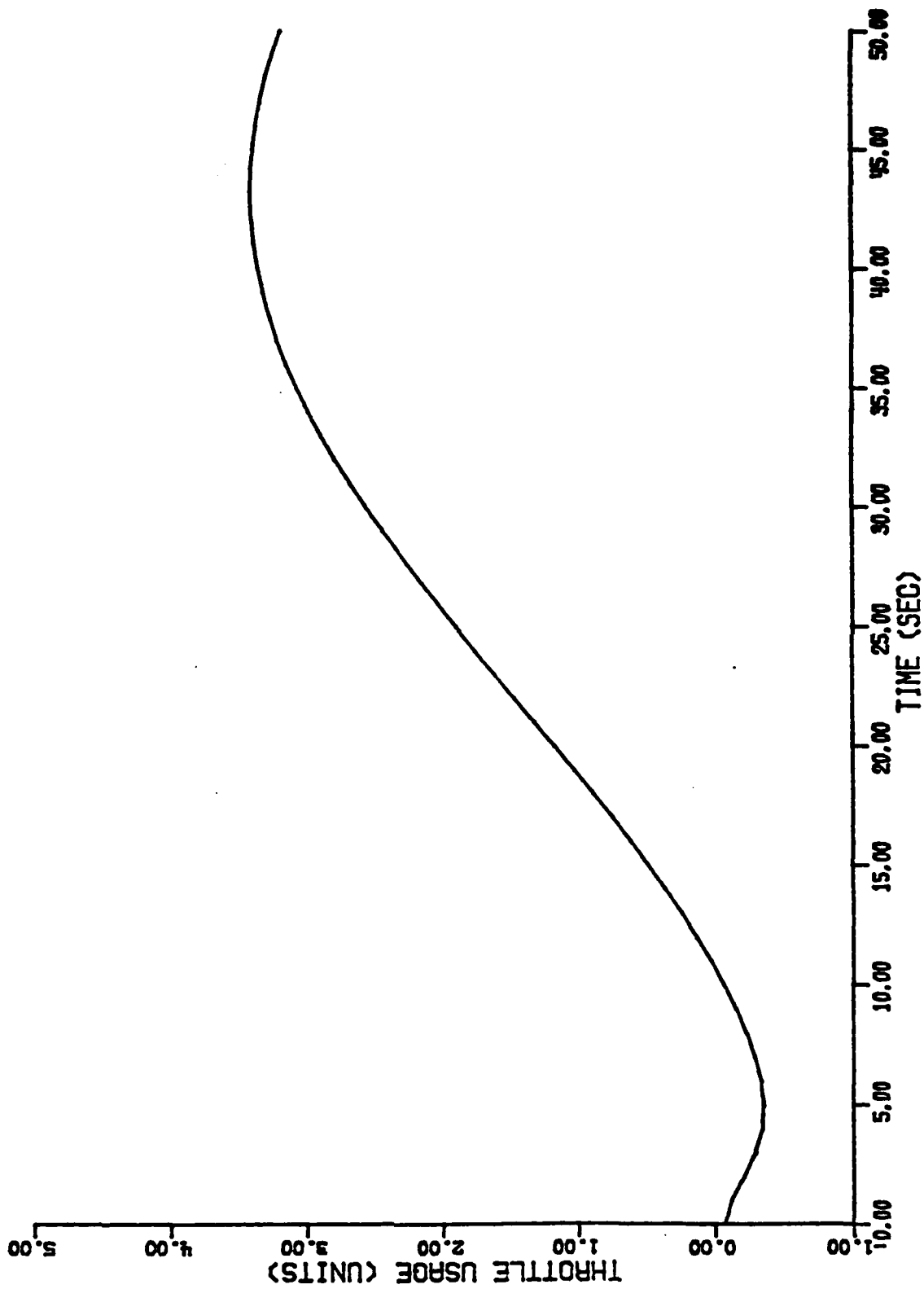


Figure 21: C-131/SST Throttle Position with MFIC

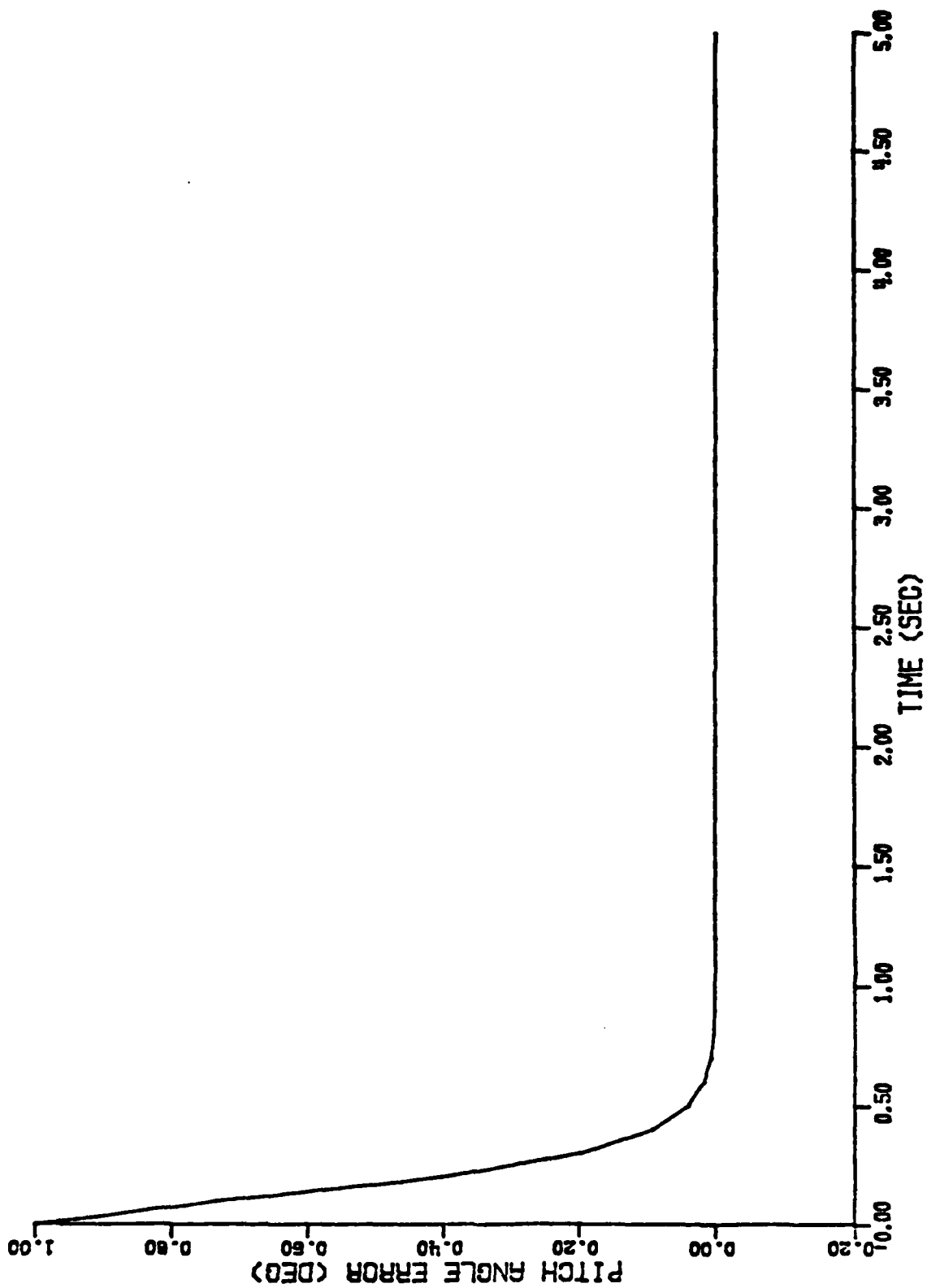


Figure 22: C-131/SST Error I.C. Response

4.8 LIMITATIONS

The Model Following for Insensitive Control design method is based on the beneficial effects of increased feedback gains on system sensitivity. As a result, it is only practical for those systems which have sufficient control authority and control bandwidth to allow the plant to be made "well-damped" and "substantially faster" than the desired final system characteristics.

The problems studied here have all assumed full-state feedback. As has been shown, many such systems have quite different responses and even instabilities in the presence of parameter uncertainties. Another large class of control-system sensitivity problems has to do with the estimator design. Some work has been done in this area (see for example [9]), with the basic conclusion being that greater insensitivity can be achieved by speeding up some of the estimator roots. The penalty for doing this is a decrease in the filtering action of the estimator, thus resulting in a noisier output.

Only uncertainties in the dynamics matrix were addressed in this research. No attempt has been made to desensitize systems to errors in the control matrix, and additional work is needed in this area.

Chapter V

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

5.1 CONCLUSIONS

Model following can be an extremely useful tool for the control-system designer. The ideas of perfect model following can provide simple, algebraically determined control laws for some problems and a useful understanding of system structure for almost any application. By combining model following with optimal control theory, the process of selecting weighting matrices is greatly simplified and a very straightforward design procedure results. Model following can also be used to design control systems which provide protection against the adverse affects of parameter uncertainties. The Model Following for Insensitive Control design method allows the designer to select the insensitivity and disturbance-rejection characteristics of the design independently of the no-disturbance, nominal-parameter performance. It takes advantage of the favorable effects of high feedback gains, and then cancels the speeded-up plant roots by using a model system as a prefilter. The method is most effective for systems in which the plant can be made to be "well-damped" and "substantially faster" than the responses required by the model to be followed.

5.2 RECOMMENDATIONS FOR FUTURE RESEARCH

1. The parameter uncertainty problems investigated here have assumed full-state feedback; however, many control system sensitivity problems are related to the design of the estimator. Model following, and in particular the MFIC design method, should be studied for possible application to estimator, and controller-plus-estimator problems.
2. Additional research is needed on the best way to handle uncertainties in the control matrix, which is a fundamentally different problem than the one addressed here.
3. The problems examined in this research involved errors in the dynamics matrix of a system with a known structure. Model following may also provide benefits in problems where higher-order structural modes have been ignored, or are not well known. Further research should be done to investigate this possibility.

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